Torsional Oscillator. Part1.

Physics 401. Spring 2019

Eugene V. Colla

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Transients in a Torsional Oscillator

- Electrical RLC circuits
- Torsional Oscillator
- Damping
- Data Analysis

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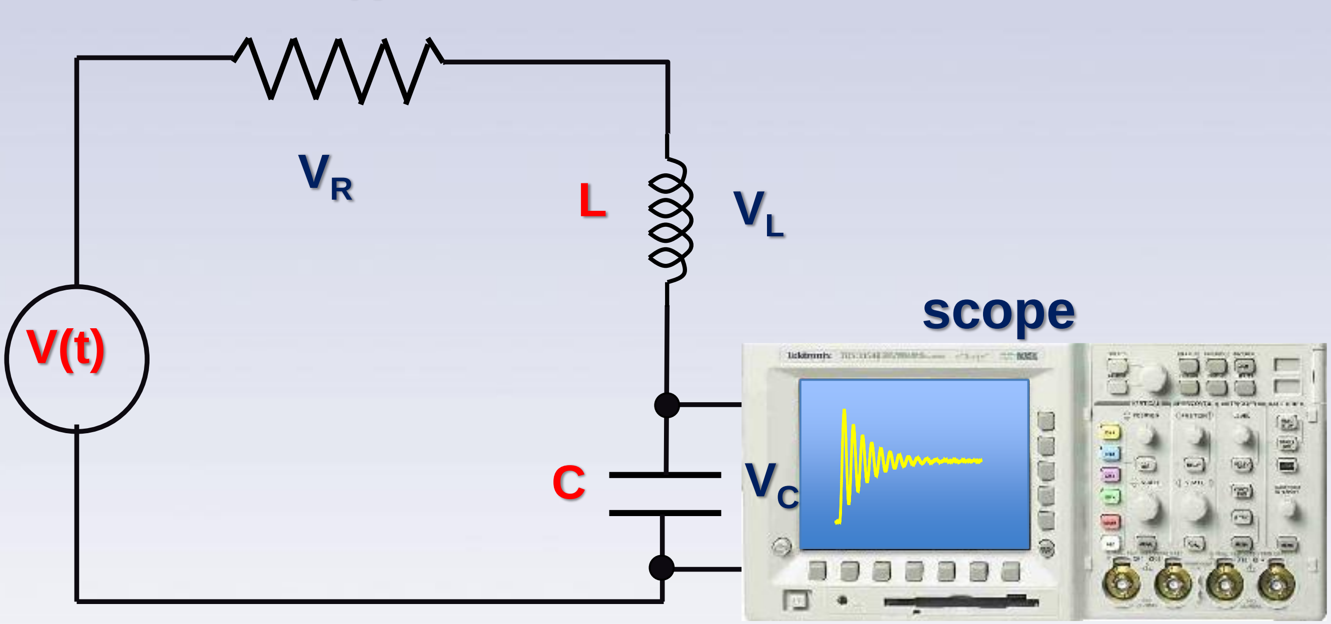
Transients in RLC circuit.

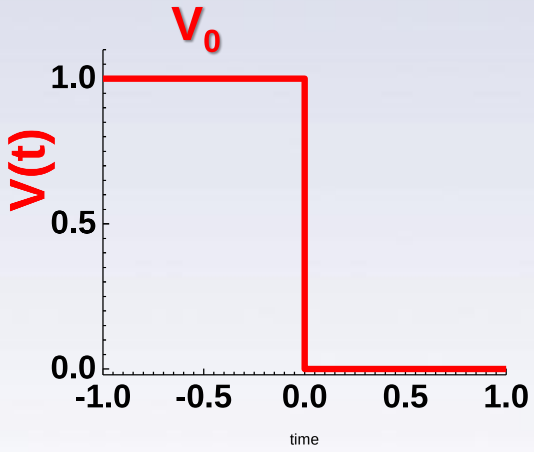
$V_R+V_L+V_C=V(t)$

If $V(t)=0$

$$L\frac{d^2}{dt^2}q(t)+R\frac{d}{dt}q(t)+\frac{q(t)}{C}=0\ ,\quad \frac{q(t)}{C}=V_0$$

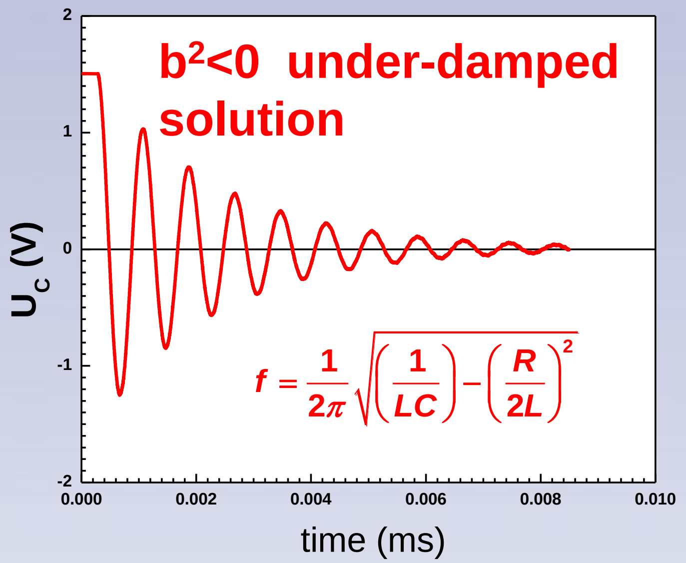
Damping term. Reflects energy dissipation in the resistor





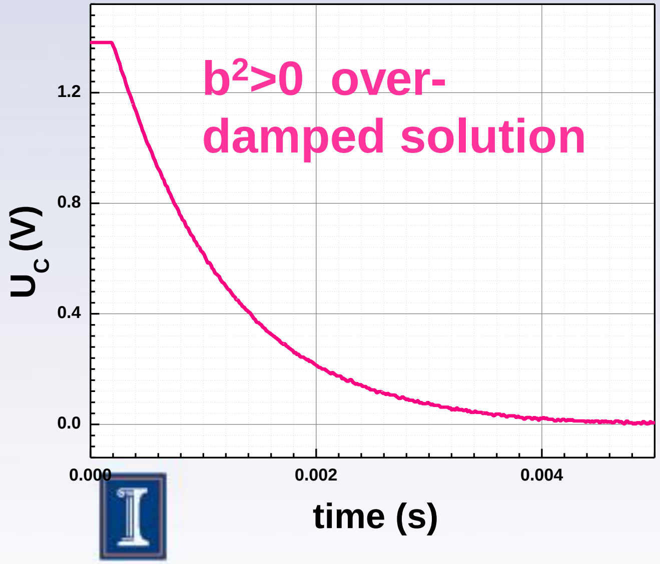
RLC: three solutions.

$$a = \frac{R}{2L} \quad , \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

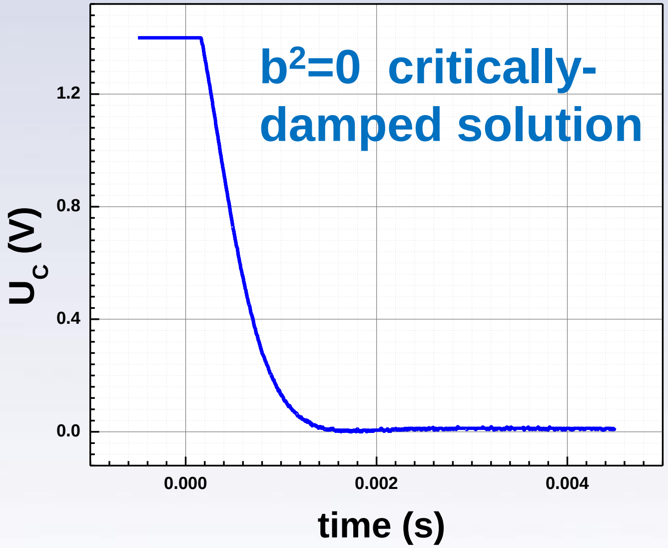


$b^2<0$ under-damped solution

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$$

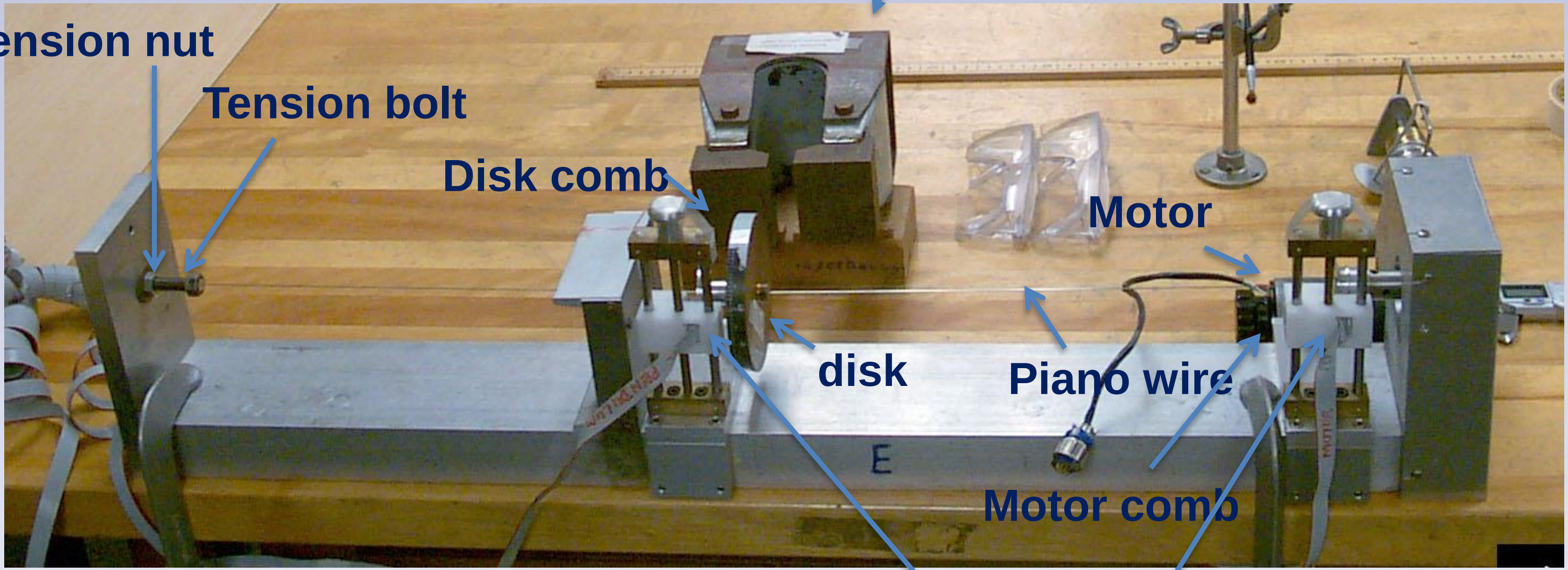


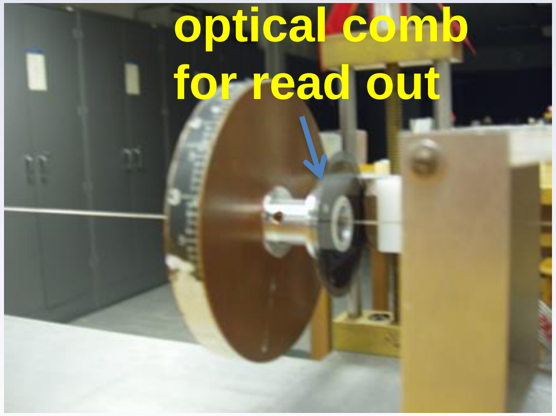
$b^2>0$ over-damped solution



$b^2=0$ critically-damped solution

The Torsional Oscillator.

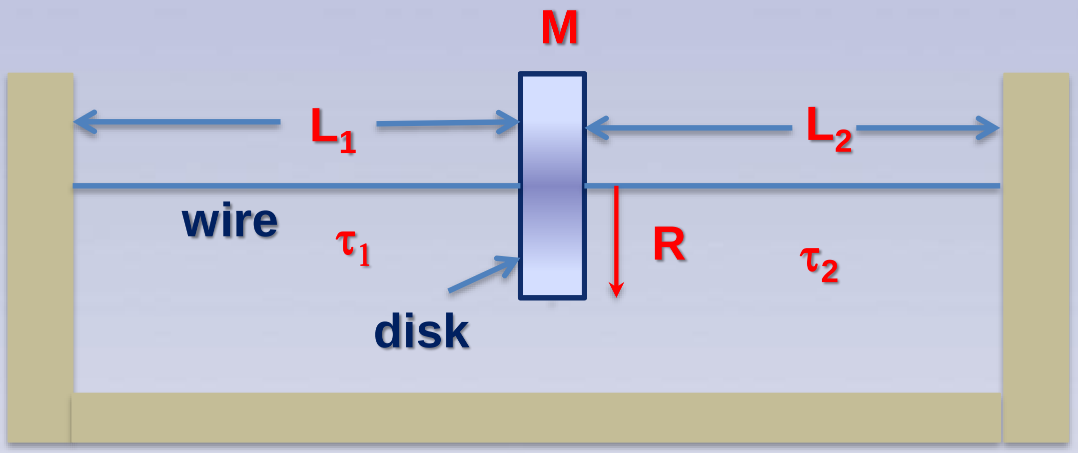




Momentum of Inertia I for disk with radius R and mass M:

$$I = \frac{MR^2}{2}$$

The Torsional Oscillator.



Wires 1 and 2 exert the torques τ_1 and τ_2 on the disk of mass M

$$\boxed{\tau = \tau_1 + \tau_2 = -K_1\theta - K_2\theta = -K\theta}$$

$$\boxed{K_1 = \frac{\pi G r^4}{2L_1}}$$

θ* : *angular deflection of the disk
r* : *radius of the wires
L_i*: *length of the wire I
G*: *shear modulus of the wire

A typical shear modulus for steel is 8.3 × 10^{10} N/m^2

$$\boxed{K = K_1 + K_2 = G\frac{\pi}{2}r^4\left(\frac{1}{L_1} + \frac{1}{L_2}\right)}$$

K* – *torsional spring constant

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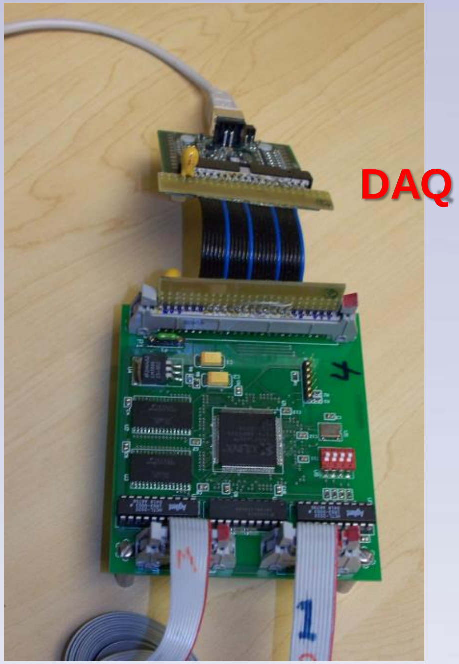
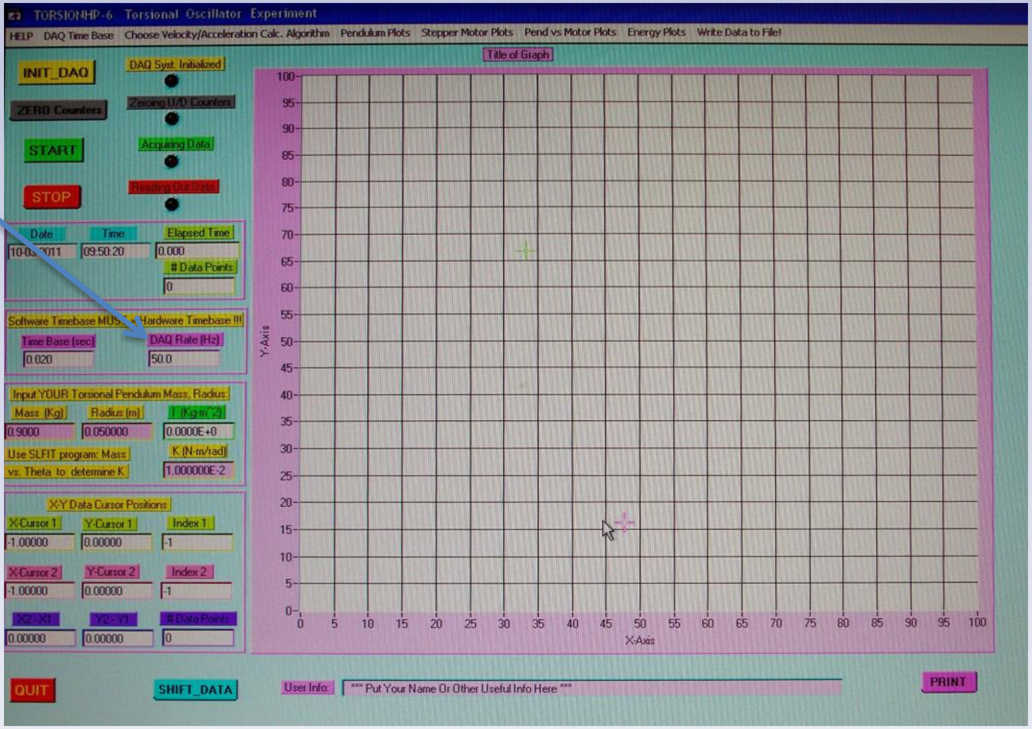
The Data Acquisition Setup and Program



DAQ rate (Hz)



Date: 10-0[illegible]2011
Time: 09:50:20
Elapsed Time: 0.000
Data Points: 0

Software Timebase MUS[illegible] Hardware Timebase !!!

Time Base (sec): 0.020
DAQ Rate (Hz): 50.0

Input YOUR Torsional Pendulum Mass, Radius:

Mass (Kg): 0.9000
Radius (m): 0.050000
I (Kg-m^2): 0.0000E+0

Use SLFIT program: Mass vs. Theta to determine K

K (N-m/rad): 1.000000E-2

X-Y Data Cursor Positions

X-Cursor 1: -1.00000
Y-Cursor 1: 0.00000
Index 1: -1
X-Cursor 2: -1.00000
Y-Cursor 2: 0.00000
Index 2: -1
X2 - X1: 0.00000
Y2 - Y1: 0.00000
Data Points: 0

User Info: *** Put Your Name Or Other Useful Info Here ***

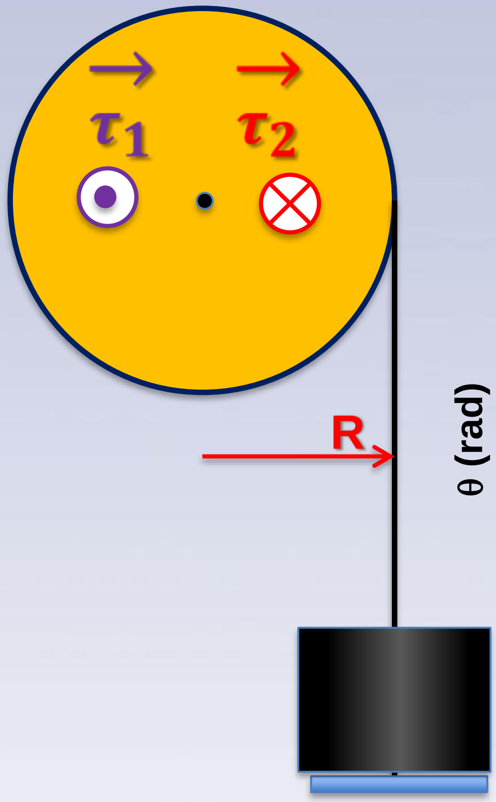
Interface card

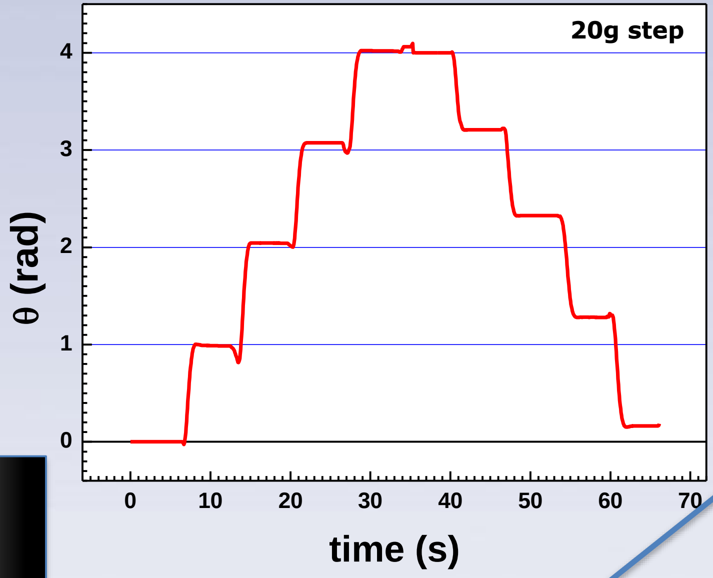
Program window

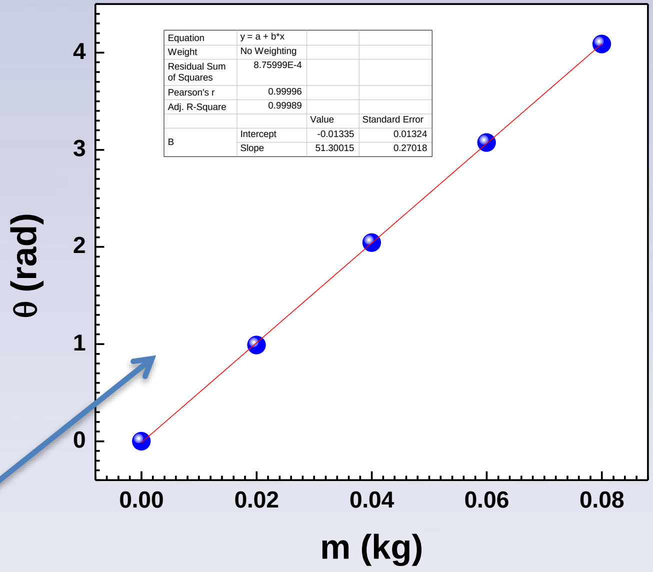
Program can accept only 10000 points. If sampling rate is 50Hz – the maximum time of data collection is 200s!

Measuring of the Torsional Spring constant.

$$\vec{\tau}_1 + \vec{\tau}_2 = 0 \longrightarrow K\theta = mgR$$







Equation	y = a + b*x		
Weight	No Weighting		
Residual Sum of Squares	8.75999E-4		
Pearson's r	0.99996		
Adj. R-Square	0.99989		
		Value	Standard Error
B	Intercept	-0.01335	0.01324
	Slope	51.30015	0.27018

slope

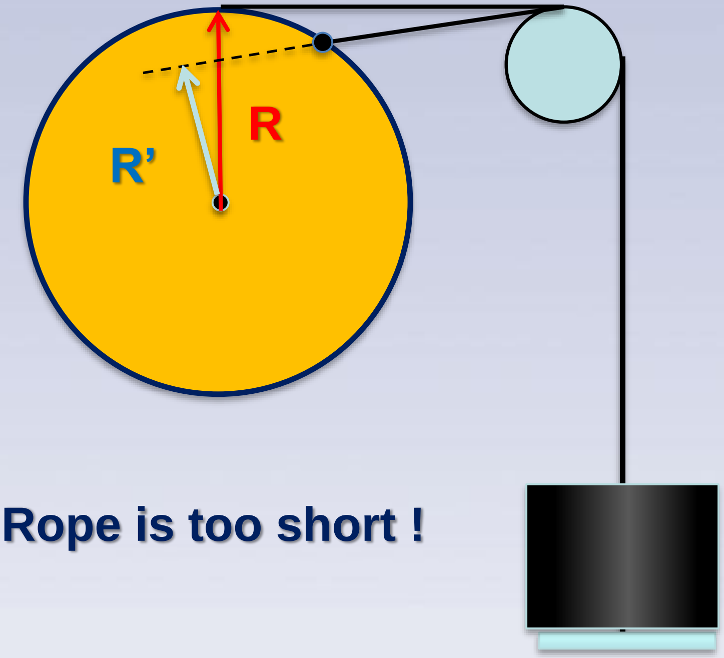
$$\theta = \boxed{\frac{gR}{K}}\, m \qquad K = \frac{gR}{slope}$$

$g=9.81 m/s^2$

Slope=51.3rad/kg

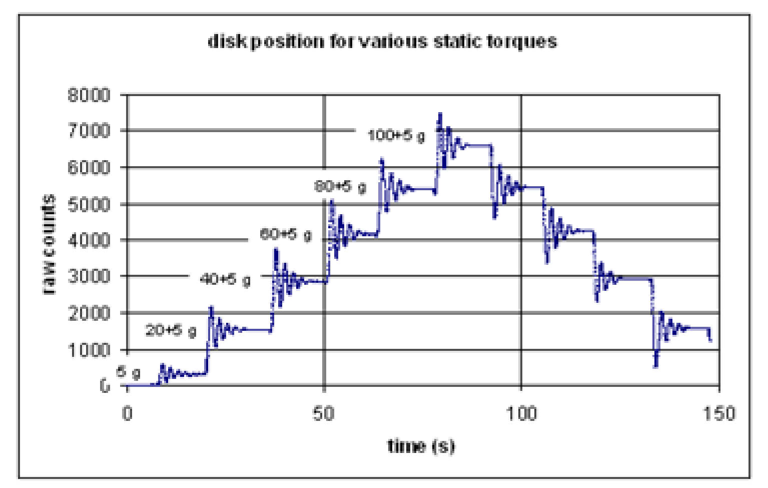
K=0.00971Nm/rad

Measuring of the Torsional Spring constant. Possible problems.



Rope is too short !

$$\tau = R \times F$$



Avoid the over damping of the pendulum motion and any extra sources of friction.

Torsional Pendulum. Scientific application.

Charles-Augustin de Coulomb
1736-1806

Measuring of the electrostatic forces.

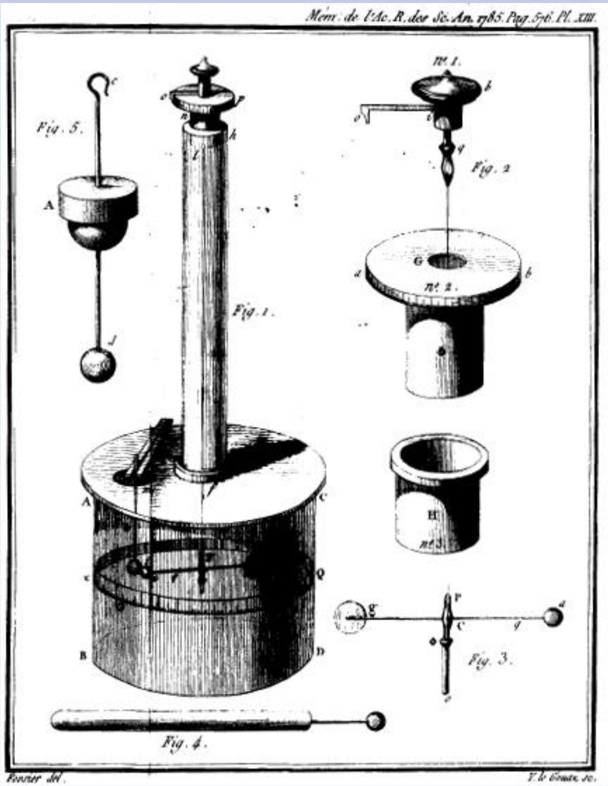
$$\vec{\tau}_1 + \vec{\tau}_2 = 0$$

$$K\theta = FL;$$

Where F is electrostatic force and L is the length of the balance beam.

Coulomb's law

$$F = k_e \frac{q1 * q2}{r^2}; \quad k_e = \frac{1}{4\pi\varepsilon_0}$$



Coulomb's torsion balance.
Courtesy of Wikipedia

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Torsional Pendulum. Scientific application.

Henry Cavendish
(1731–1810)

Measuring of the gravitational forces.

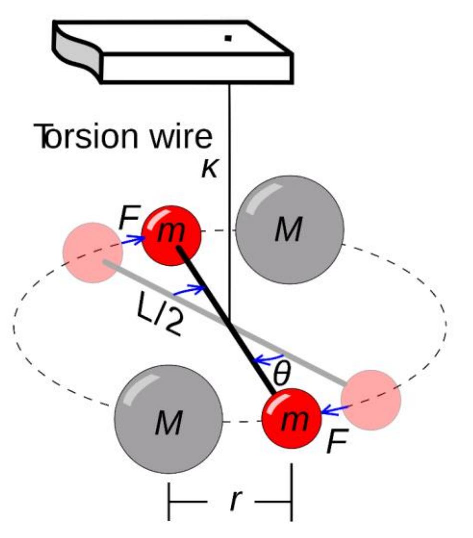
$$\vec{\tau}_1 + \vec{\tau}_2 = 0$$

$$K\theta = FL;$$

Where F is gravitational force and L is the length of the balance beam.

Gravitational Law

$$\boxed{F = \frac{GmM}{r^2}}$$



Cavendish torsion balance experiment.

Courtesy of Wikipedia

Cavendish's result

$G = 6.74 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$,

Currently accepted value

$6.67428 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$.

The Torsional Oscillator. “No damping”.

$$\tau=\tau_1+\tau_2=-K_1\theta-K_2\theta=-K\theta$$

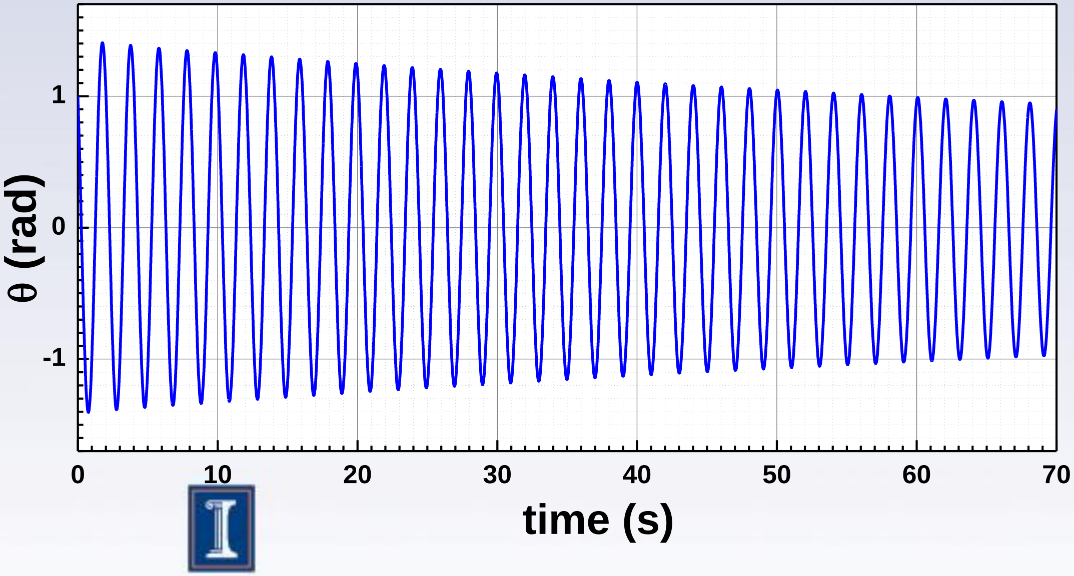
$$K_1=\frac{\pi G r^4}{2L_1};\quad K=K_1+K_2=\frac{\pi G r^4}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)$$

If there is no dissipation:

$$I\frac{d^2\theta}{dt^2}=-K\theta$$

Solution: $\theta=\theta_0\sin(\omega_0 t+\phi)$ **with** $\omega_0=\sqrt{\frac{K}{I}}$

If we know I **we can calculate** K



From time trace $\theta(t)$ **we can find** ω_0 **it can be done by measuring period but better (and faster!) to perform the nonlinear fitting.**

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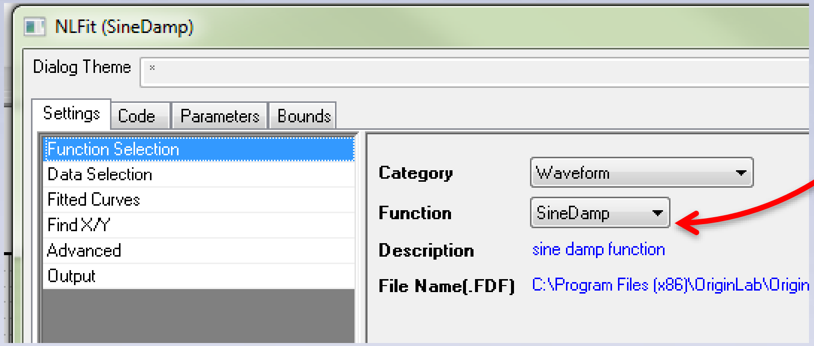
The Torsional Oscillator. "No damping". Fitting

"No damping" is not realistic situation fitting should be done to

SineDamp function $y = y_0 + A\exp\left(\frac{-x}{t_0}\right)\sin\left(\pi\frac{(x-x_c)}{w}\right)$ $\quad \omega_0 = \frac{\pi}{w}$


NLFit (SineDamp)

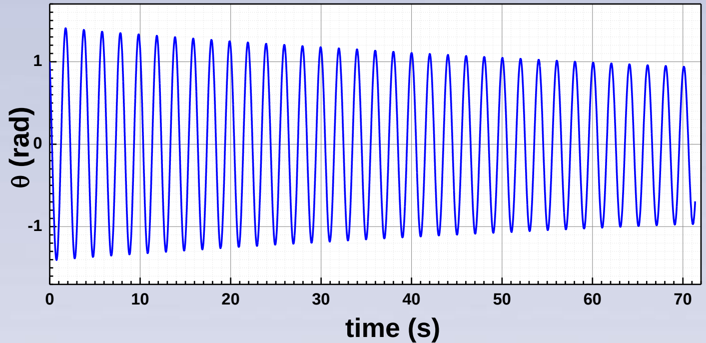
Dialog Theme

Category Waveform

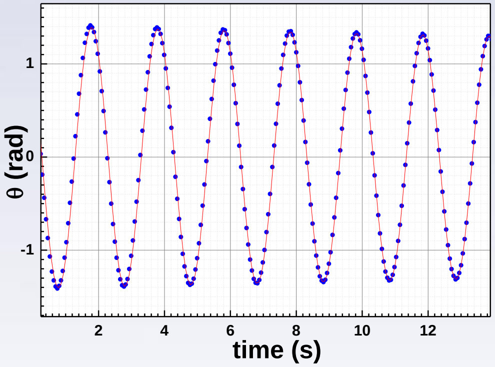
Function SineDamp

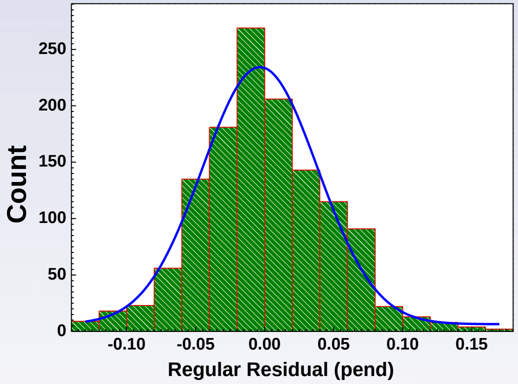
Description sine damp function

File Name(.FDF) C:\Program Files (x86)\OriginLab\Origin



$$\omega_0 = 3.126\frac{rad}{s};\ f_0 = \frac{\omega_0}{2\pi} \approx 0.4975\,Hz$$





SineDamp: $y = y_0 + A\exp\left(\frac{-x}{t_0}\right)\sin\left(\pi\frac{(x-x_c)}{w}\right)$

	Value	Standard Error
y0	-0.0024	0.0013
xc	-0.7236	9.3E-4
w	1.00517	2.5E-5
t0	178.02	2.44
A	1.409	0.004

$$K = \omega_0{}^2 I \approx 1.12 \times 10^{-2}\frac{Nm}{rad}$$

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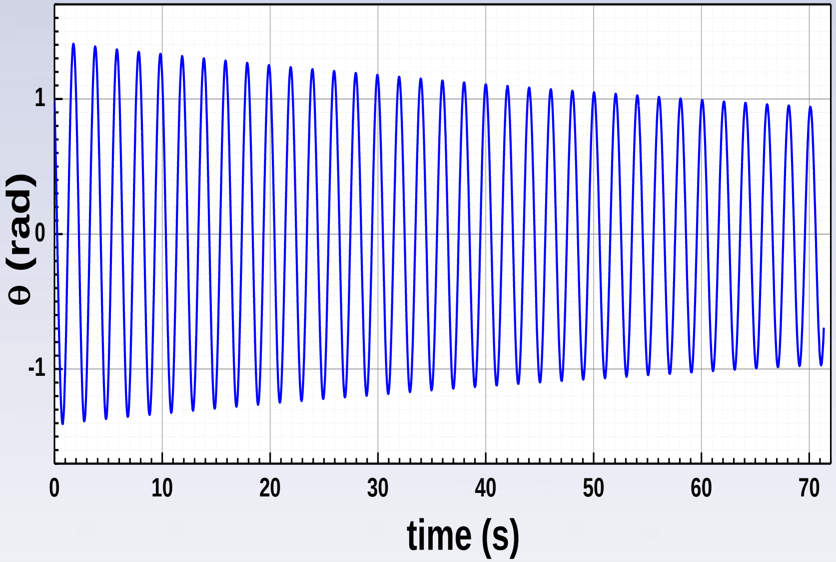
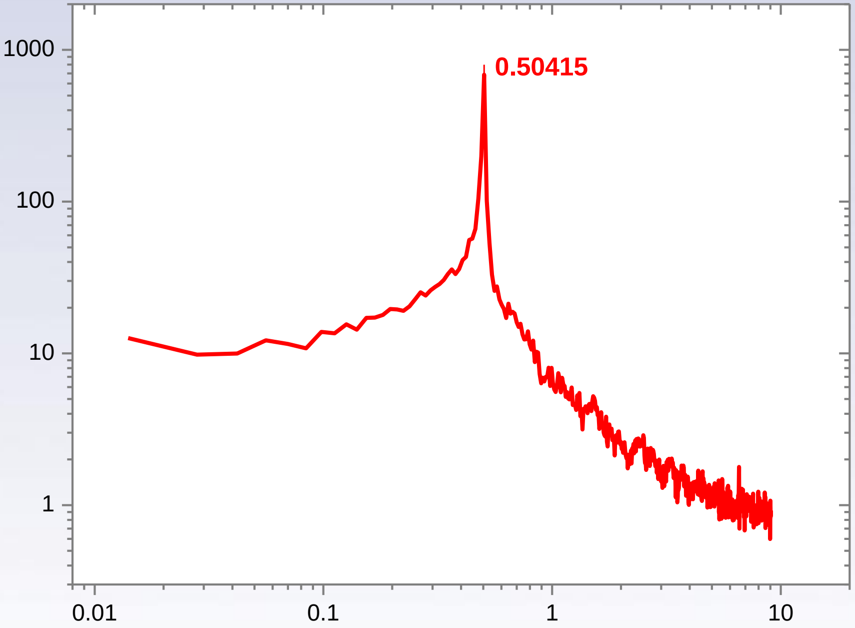
The Torsional Oscillator. "No damping". Fitting.

$$\omega_0 = \frac{\pi}{w} \qquad f_0 = \frac{1}{2w}$$

From "SineDamp fitting f_0=0.497Hz
or $\omega_0 = 2\pi f_0 = 3.123 rad/s$

Resonance frequency can be also found by applying FFT on the raw data





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Three different kinds of mechanisms.

1. Viscous (magnetic) damping
2. Coulomb damping
3. Turbulent damping

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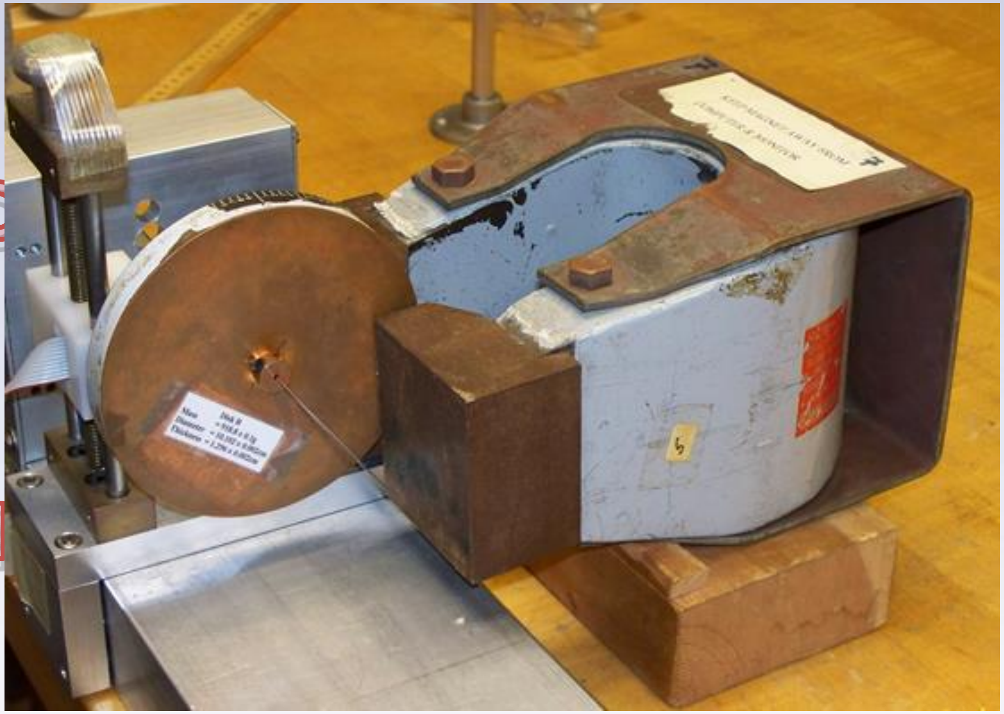
Three different kinds of mechanisms.

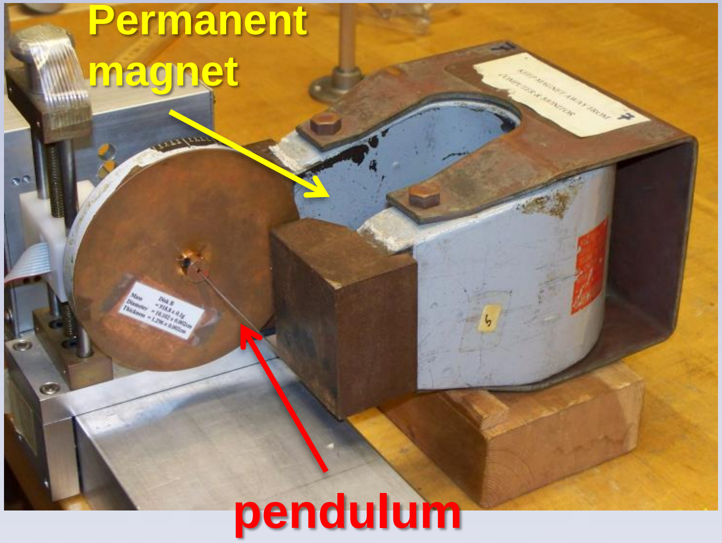
1. Viscous (magnetic) damping

2.Co

3.Tu



Viscous (magnetic) damping.

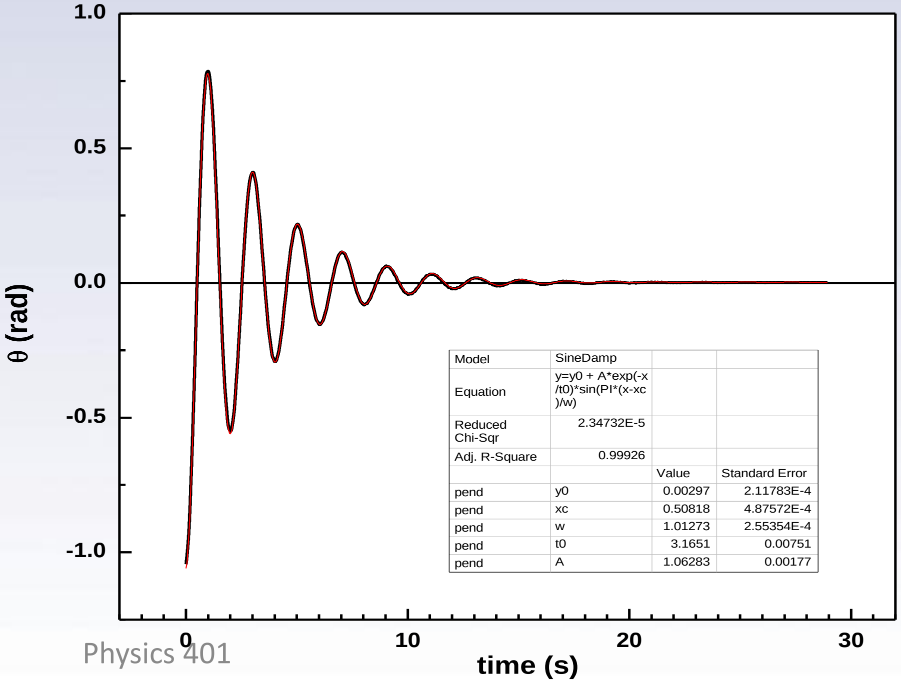


$$I\frac{d^2\theta}{dt^2}+K\theta+R\frac{d\theta}{dt}=0$$

Damping term

The solutions are exactly the same as in case of RLC circuit (three solutions)

Under damped case



Model	SineDamp		
Equation	y=y0 + A*exp(-x/t0)*sin(PI*(x-xc)/w)		
Reduced Chi-Sqr	2.34732E-5		
Adj. R-Square	0.99926		
		Value	Standard Error
pend	y0	0.00297	2.11783E-4
pend	xc	0.50818	4.87572E-4
pend	w	1.01273	2.55354E-4
pend	t0	3.1651	0.00751
pend	A	1.06283	0.00177

Viscous damping. Logarithmic decrement.

$$I\frac{d^2\theta}{dt^2} + K\theta + R\frac{d\theta}{dt} = 0$$

$$\delta = ln\left(\frac{\theta_{n+1}}{\theta_n}\right);$$

For viscous damping $\delta = \frac{T}{t_0}$;

where

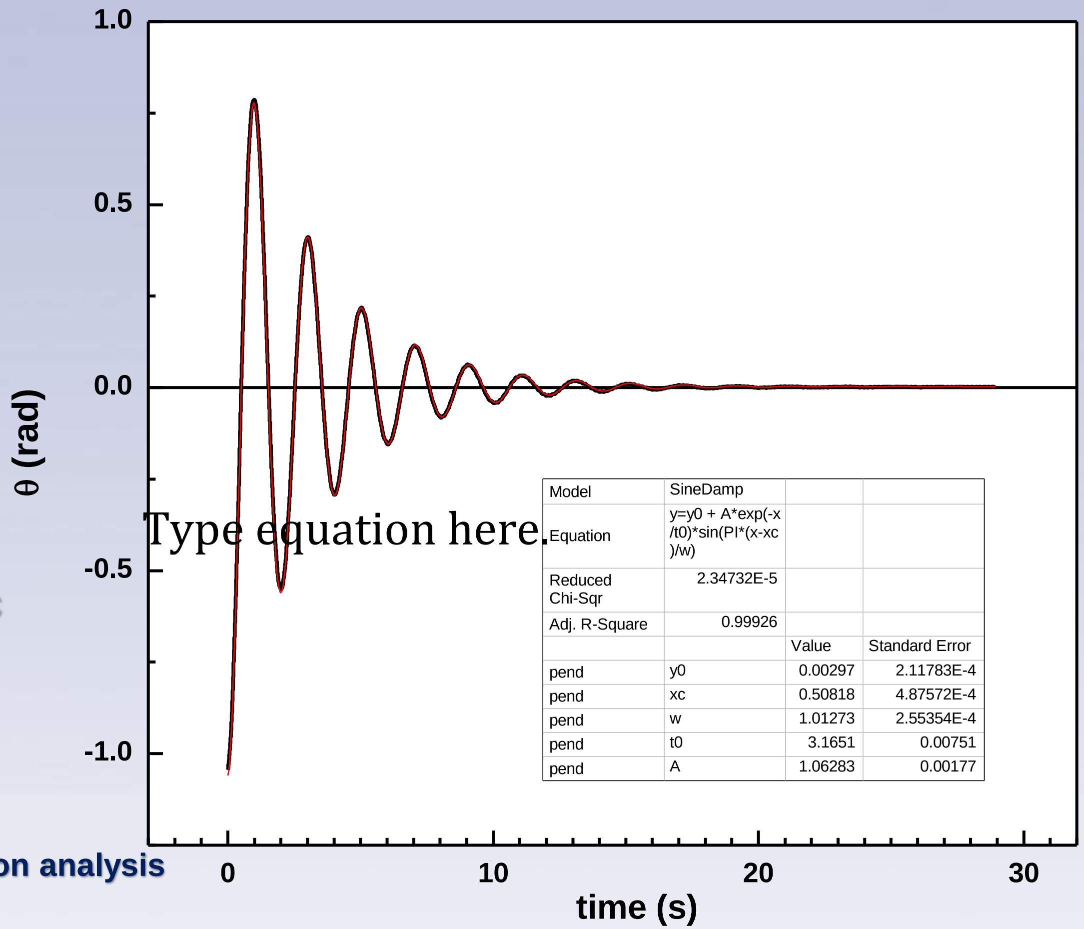
T – period and t_0 – characteristic decay time*

from SineDamp fitting function

T=2w and $\delta = \frac{2w}{t_0}$

δ=0.640±0.002

from error propagation analysis



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pend	w	1.01273	2.55354E-4
pend	t0	3.1651	0.00751
pend	A	1.06283	0.00177

Type equation here.

From *SineDamp* fitting exponential decay term is $\exp\left(\frac{-t}{t_0}\right)$

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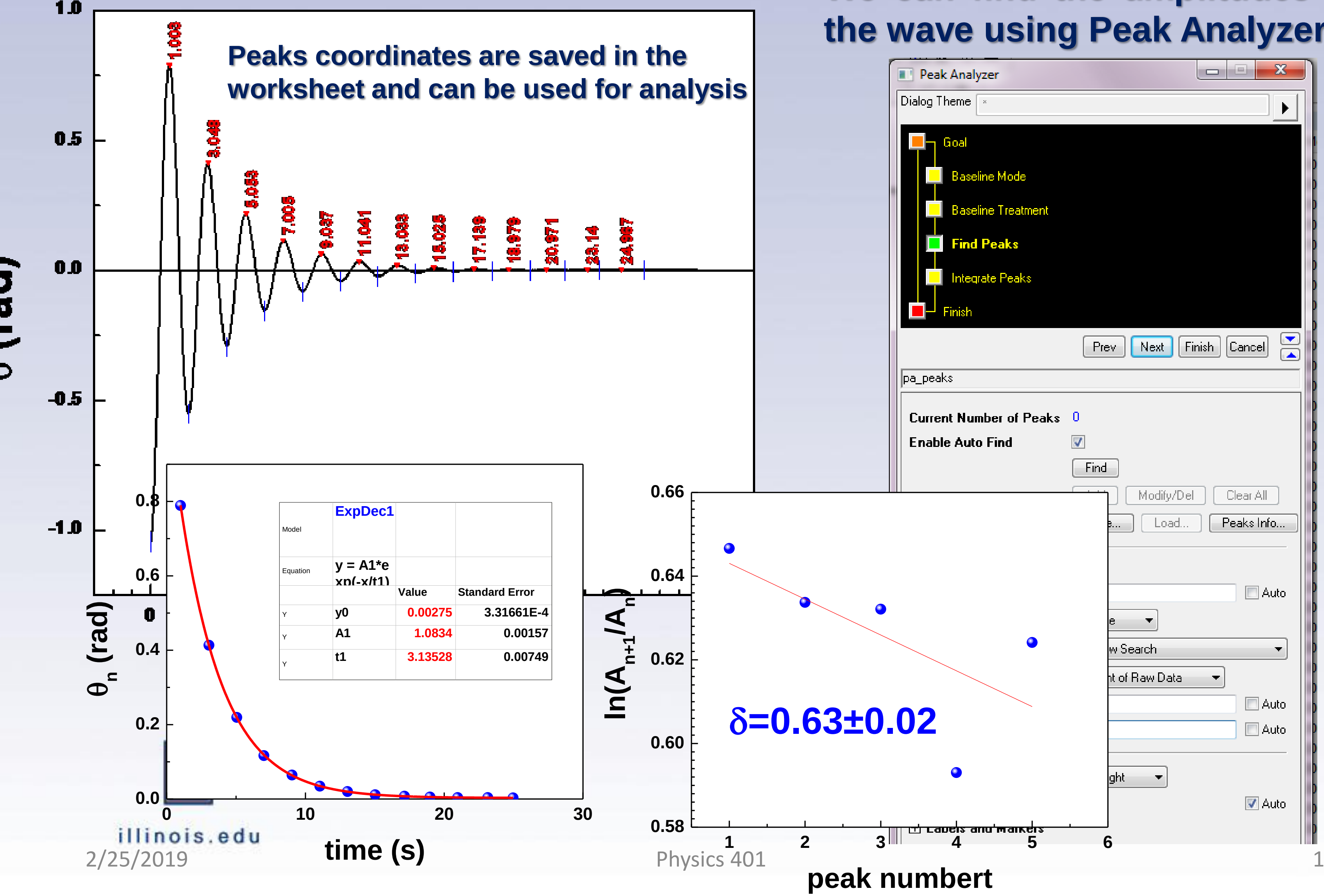
$*a=\frac{1}{t_0}$ (write up)

Viscous damping. Logarithmic decrement.

We can find the amplitudes of the wave using Peak Analyzer



Peaks coordinates are saved in the worksheet and can be used for analysis

Model	ExpDec1		
Equation	y = A1*exp(-x/t1)		
		Value	Standard Error
Y	y0	0.00275	3.31661E-4
Y	A1	1.0834	0.00157
Y	t1	3.13528	0.00749

pa_peaks

Current Number of Peaks 0

Enable Auto Find ☑

δ=0.63±0.02

Analysis. 1

1. Fitting to damp exponential decay function. Outcome: resonance frequency and decrement coefficient .
2. Applying FFT procedure. Result – resonance frequency.
3. Using Origin Peak Analyzer we can find amplitudes and positions of the damped sine wave maximum end then plot the envelope.
4. You can directly obtain the envelope of the damped sine wave by using Origin (optional).

Three different kinds of mechanisms.

1. Viscous (magnetic) damping

2. **Coulomb damping**

3. Turbu g

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Coulomb damping. Theory

$$I\ddot{\theta} + K\theta + \tau_{Coulomb} = 0$$

$$\tau_{Coulomb} = C\frac{|\dot{\theta}|}{\dot{\theta}}$$

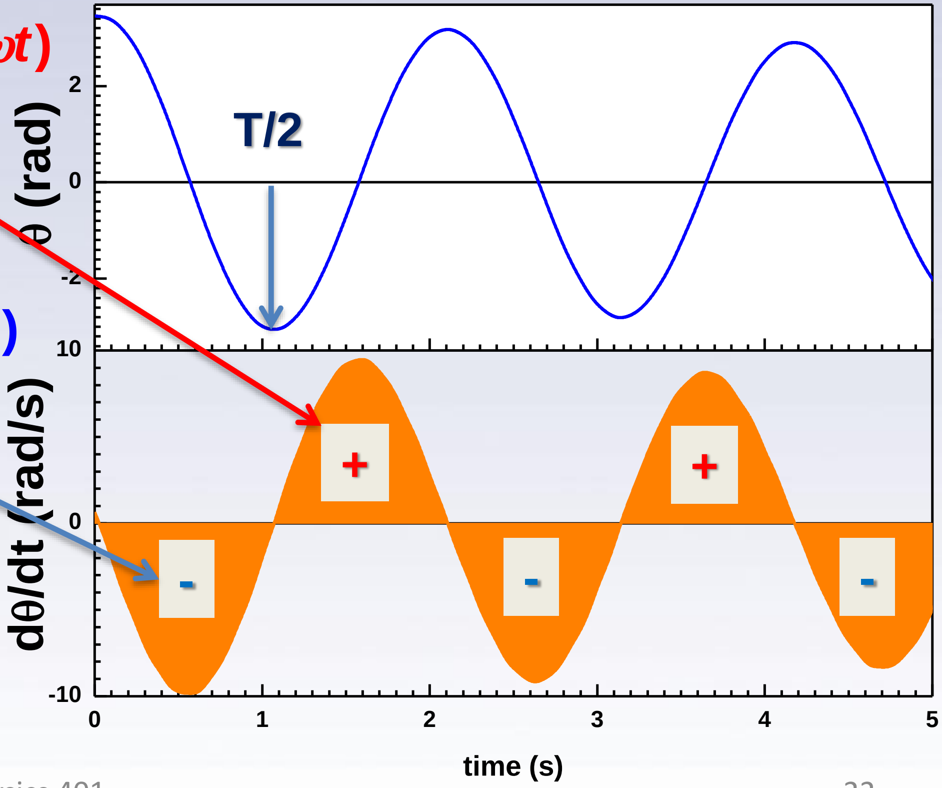
Amplitude decreases by 4C/K per period linearly !

$$\theta(t) = +C/K + (\theta_0 - (4n-1)C/K)\cos(\omega t)$$

$$(n-\frac{1}{2})T \le t \le nT \quad n = 1,2,\ldots$$

$$\theta(t) = +C/K + (\theta_0 - (4n-3)C/K)\cos(\omega t)$$

$$(n-1)T \le t \le (n-\frac{1}{2})T \quad n = 1,2,\ldots$$



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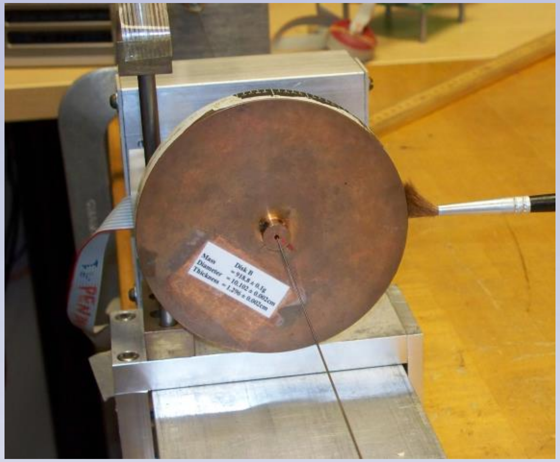
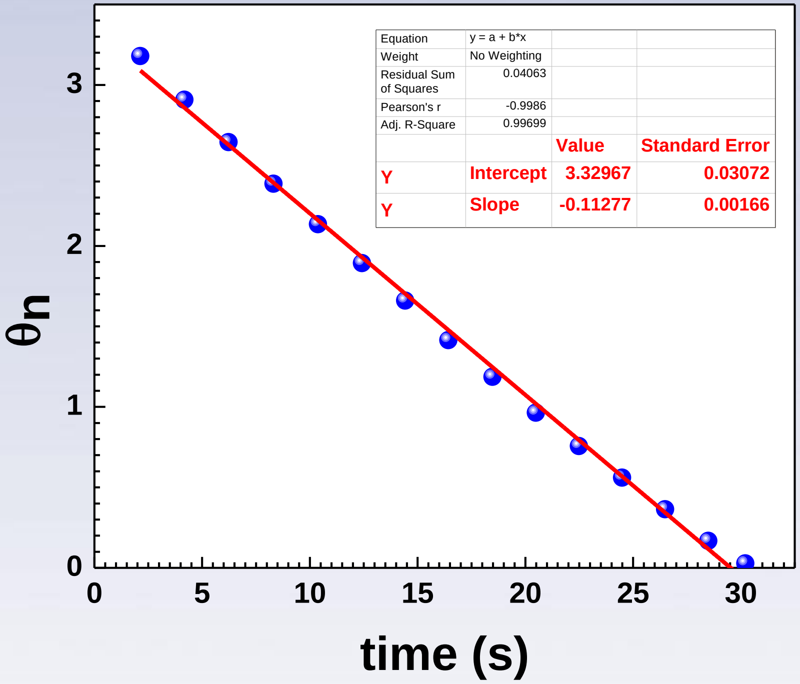
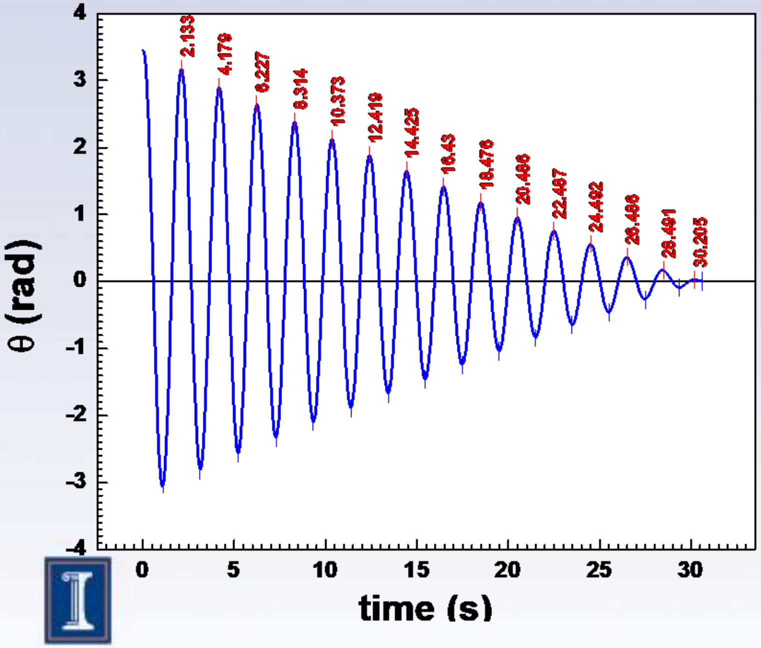
Coulomb damping. Experiment



Amplitude decreases by 4C/K per period linearly !



Equation	y = a + b*x		
Weight	No Weighting		
Residual Sum of Squares	0.04063		
Pearson's r	-0.9986		
Adj. R-Square	0.99699		
		Value	Standard Error
Y	Intercept	3.32967	0.03072
Y	Slope	-0.11277	0.00166



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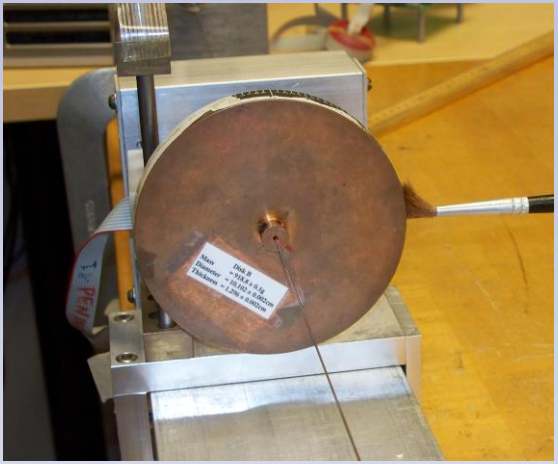
Coulomb damping. Experiment

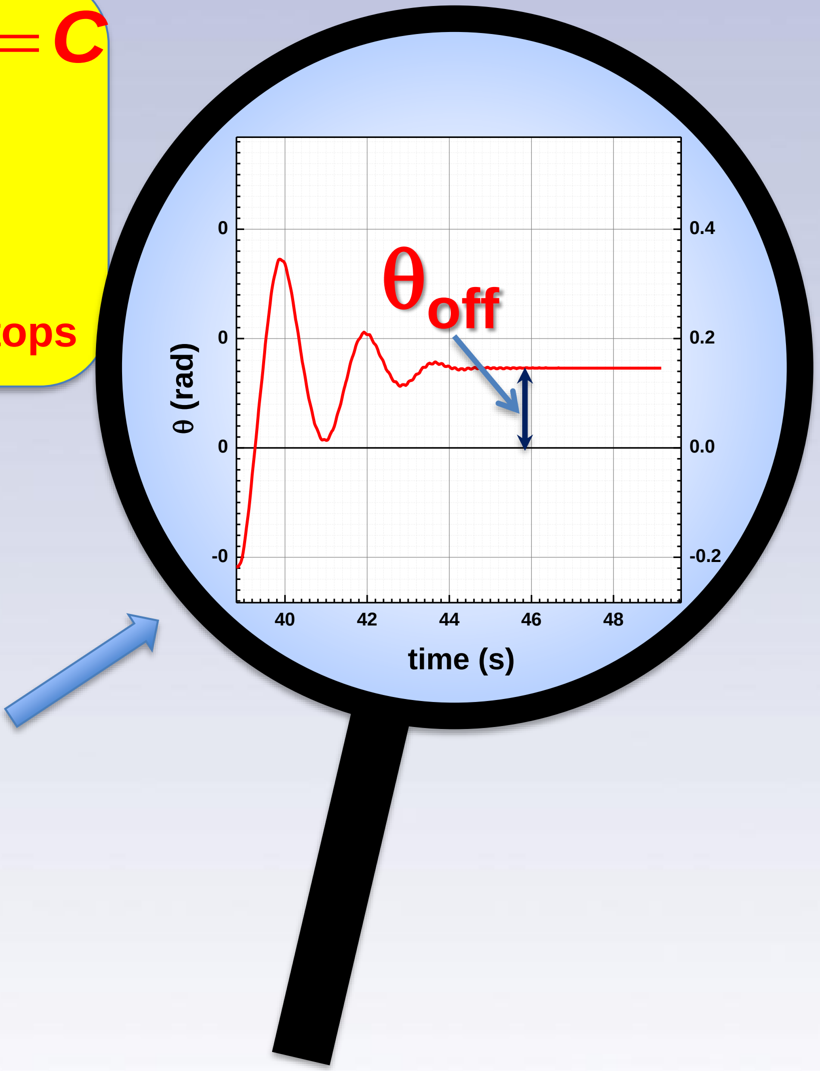


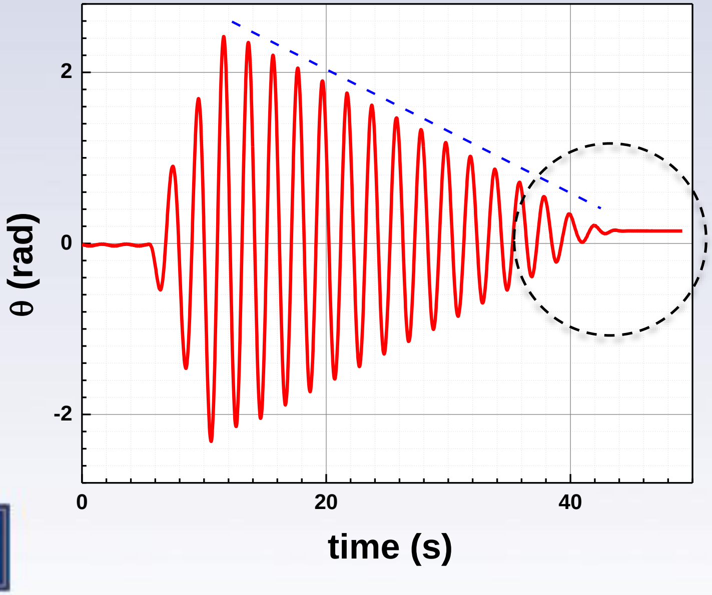
$$\left|\tau_{Coulomb}\right| = C$$

$$K\theta \sim \theta$$

if $K\theta \le C$

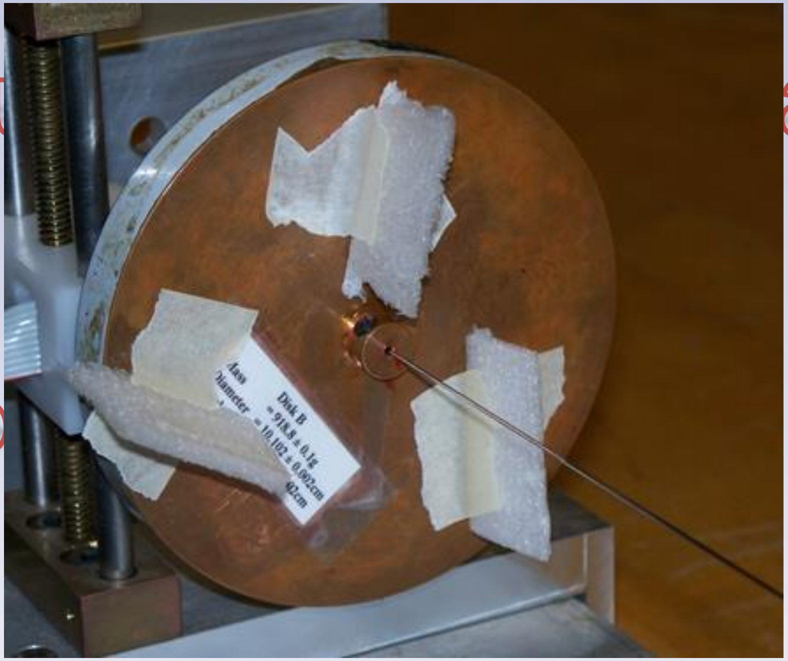
pendulum stops





Three different kinds of mechanisms.

1.Visco amping



2.Coulo

3.Turbulent damping

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Turbulent damping. Theory

$$\boxed{I\ddot{\theta}+K\theta+\tau_{Turb}=0}$$

$$\tau_{turb}=C_t\,\mathrm{sgn}(\dot{\theta})\left|\dot{\theta}\right|^n$$

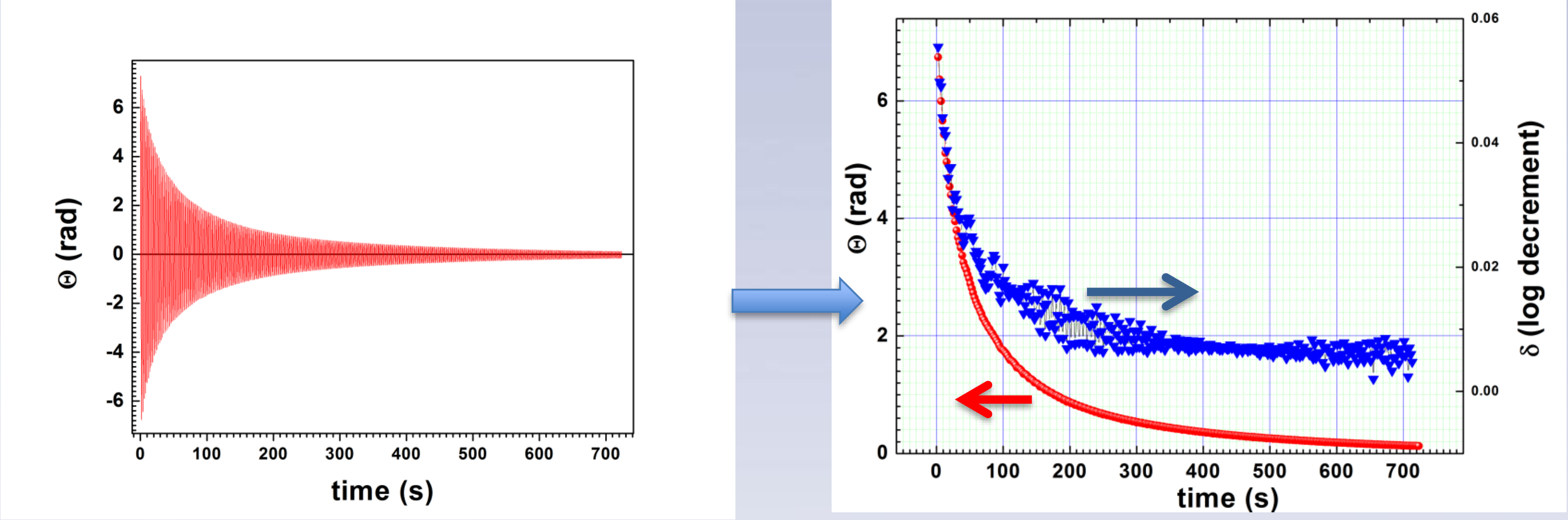
In case of n=1 → viscous damping

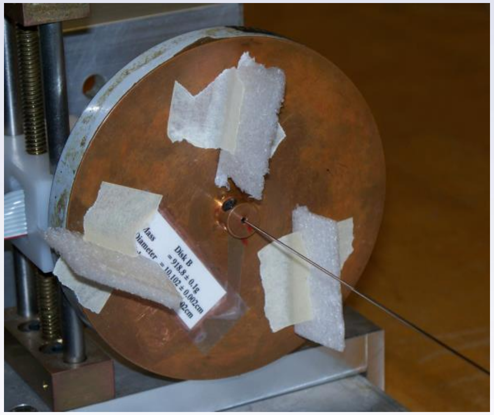
Logarithmic decrement in case of turbulent damping is no more constant and in case n=2 can be calculated as $\delta=\frac{8C}{3I}\theta_0$

Expected result – decrement decreases with decreasing of the amplitude

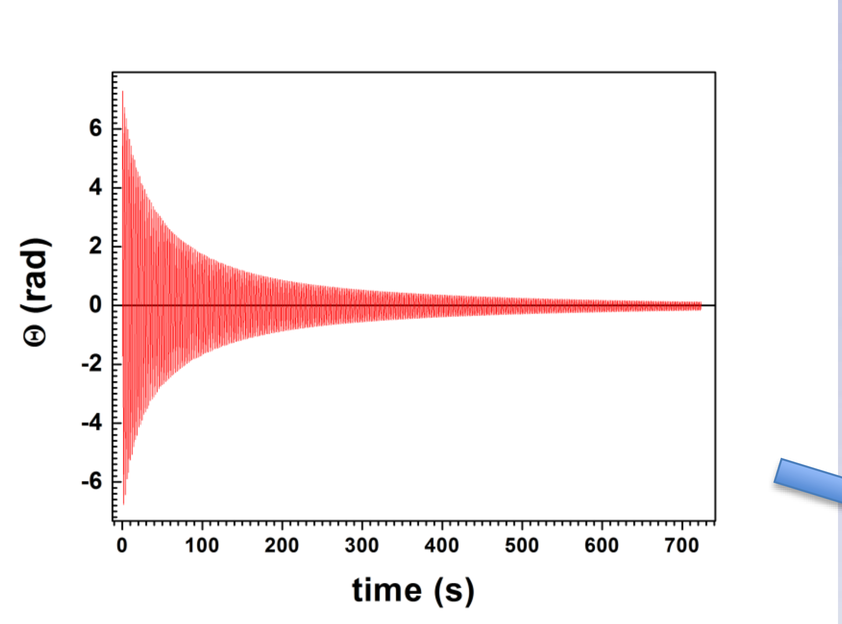
Turbulent damping. Experiment

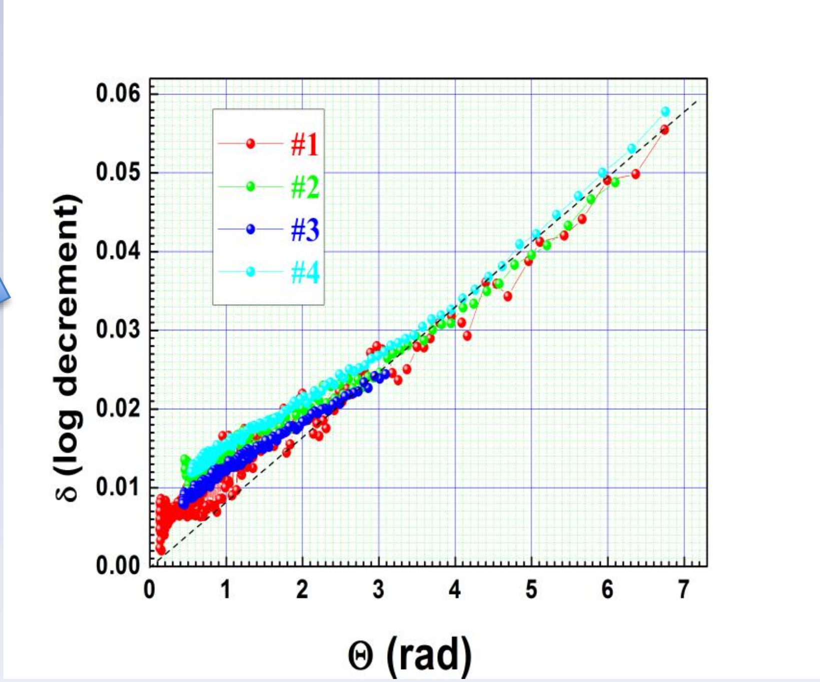
Analyzing the envelope of the damped oscillating time record we can calculate the log decrement factor

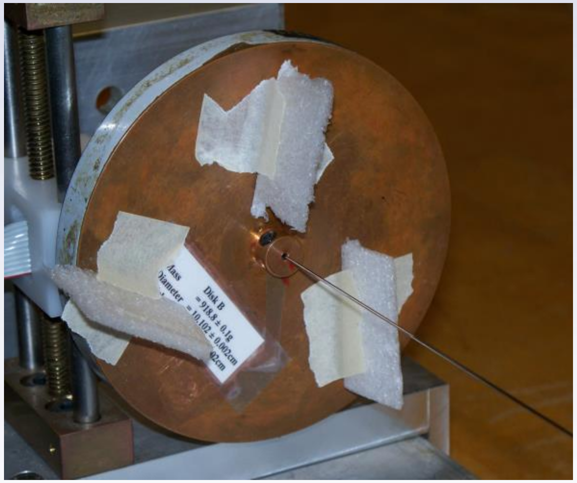




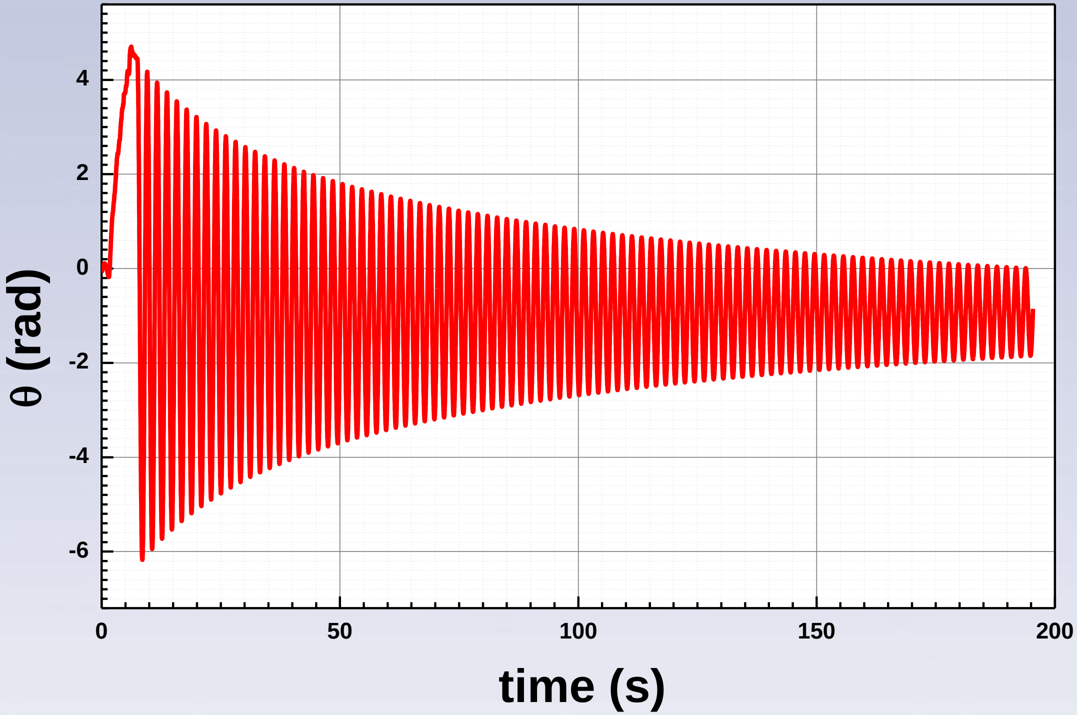
Turbulent damping. Experiment.







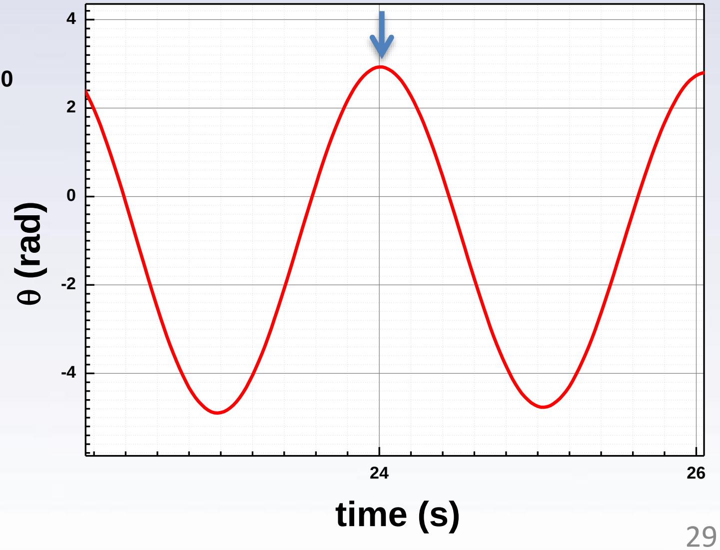
Data analysis. Finding the peaks.



Raw data.

Our goal: find the positions and amplitudes of the peaks

X_i, Y_i



1st Technique: using "*FindPeaks*" option

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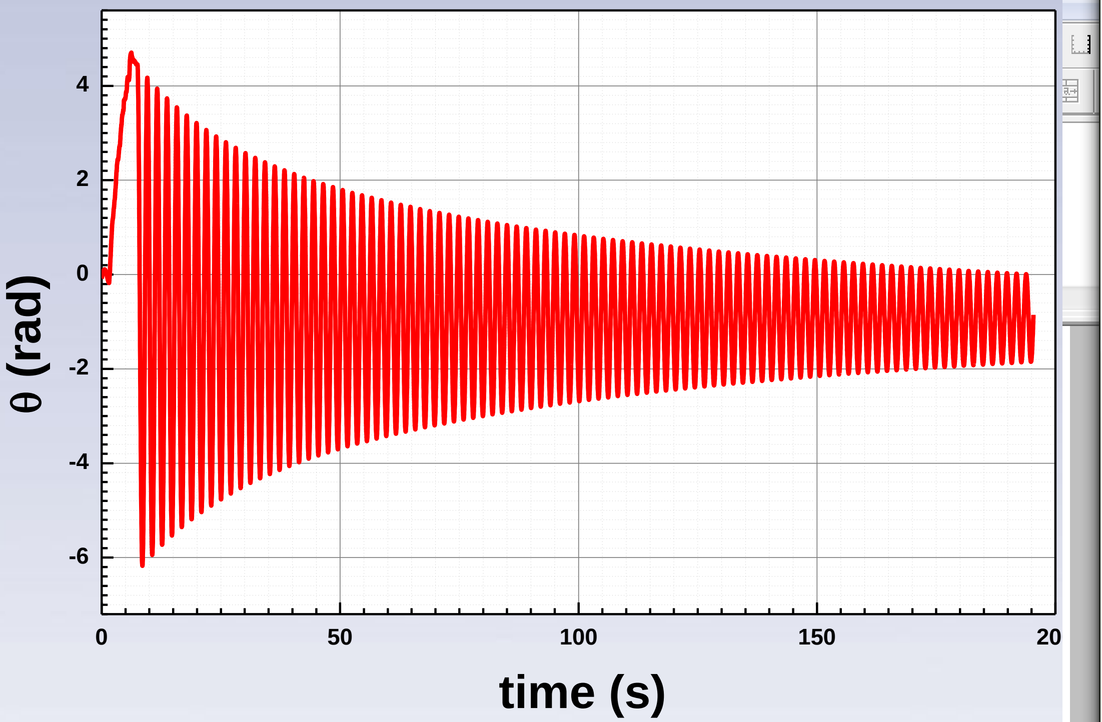
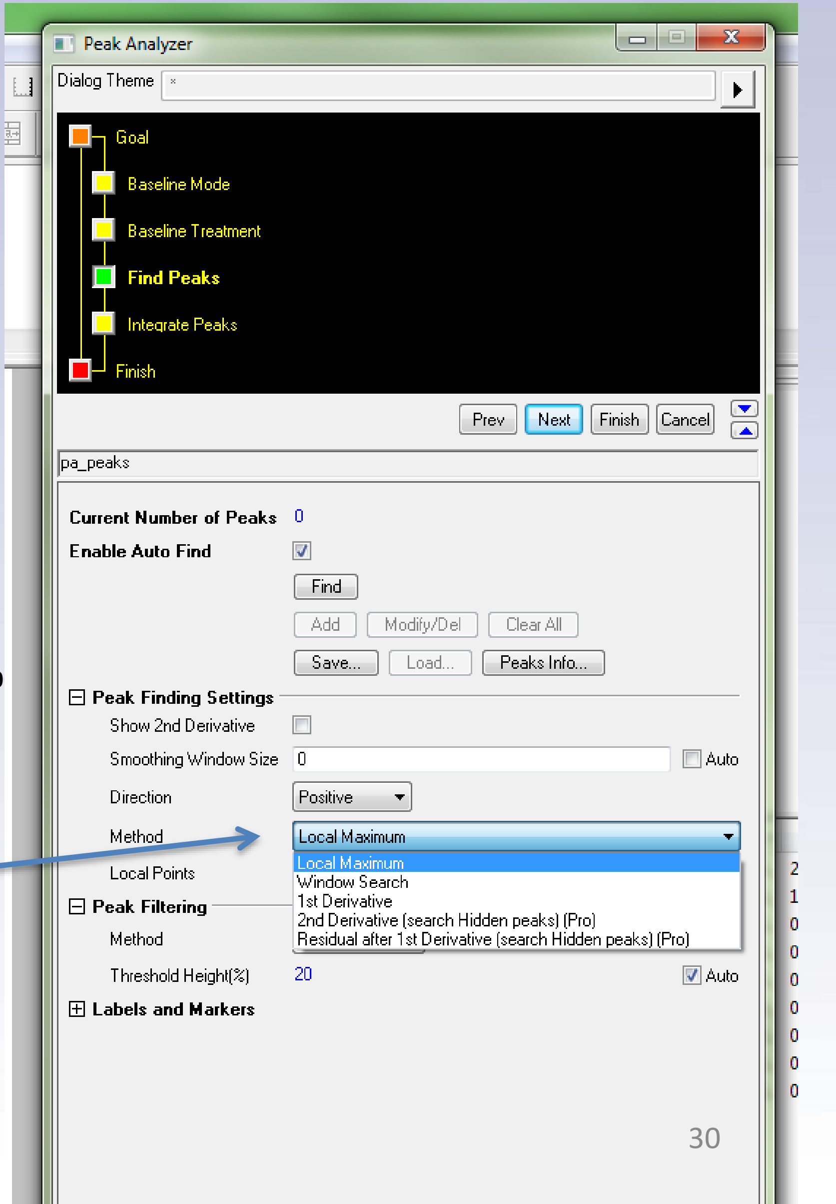
Data analysis. Finding the peaks.





pa_peaks

Current Number of Peaks 0

Enable Auto Find ☑

⊟ **Peak Finding Settings**

Show 2nd Derivative ☐

Smoothing Window Size 0 ☐ Auto

Direction Positive

Method Local Maximum

Local Points

⊟ **Peak Filtering**

Method

Threshold Height(%) 20 ☑ Auto

⊞ **Labels and Markers**

Local Maximum **works well for not noisy oscillating dependencies**

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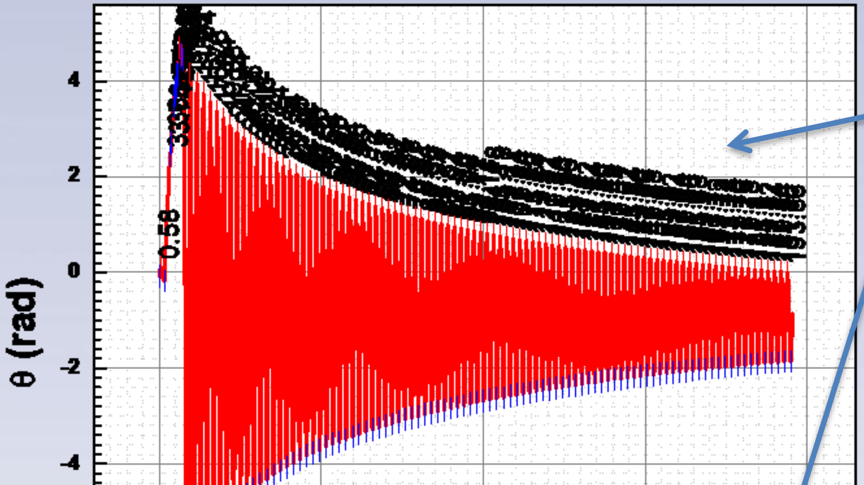
Data analysis. Finding the peaks.

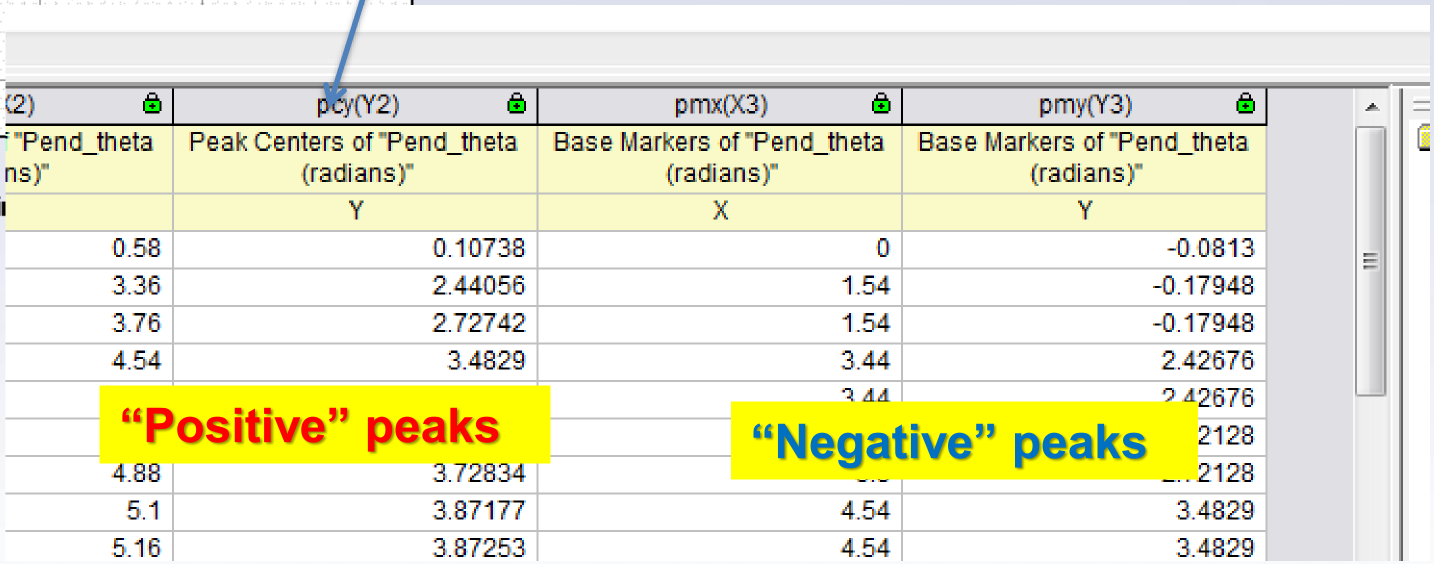
The details related to this project you can find in:

\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\6. Torsional oscillator\Turbulent damping.opj



New plot + labels as a result of finding the peaks

"Peaks" data can be found in a Worksheet and using this data you can plot the dependence of amplitude on time



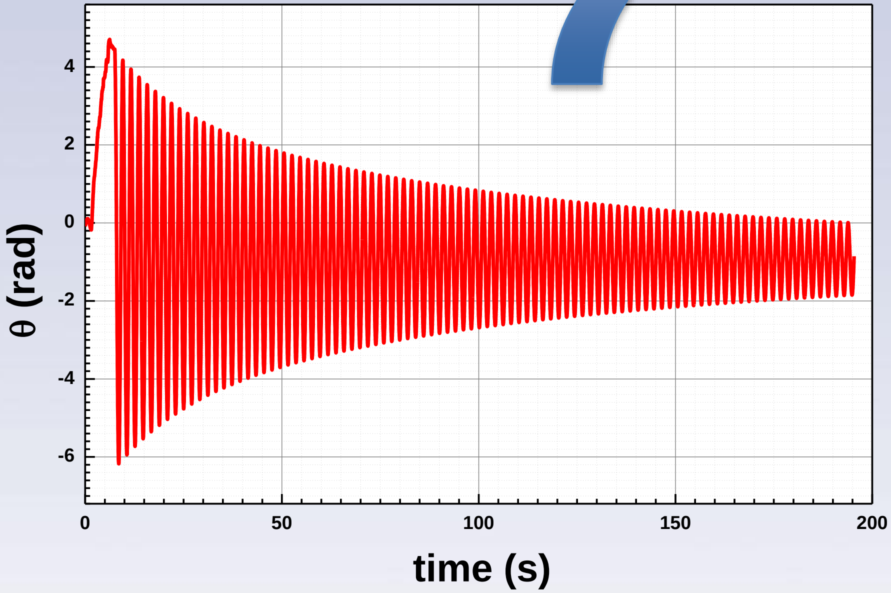
(2)	pcy(Y2)	pmx(X3)	pmy(Y3)
"Pend_theta ns)"	Peak Centers of "Pend_theta (radians)"	Base Markers of "Pend_theta (radians)"	Base Markers of "Pend_theta (radians)"
	Y	X	Y
0.58	0.10738	0	-0.0813
3.36	2.44056	1.54	-0.17948
3.76	2.72742	1.54	-0.17948
4.54	3.4829	3.44	2.42676
		3.44	2.42676
			2128
4.88	3.72834		2128
5.1	3.87177	4.54	3.4829
5.16	3.87253	4.54	3.4829

"Positive" peaks

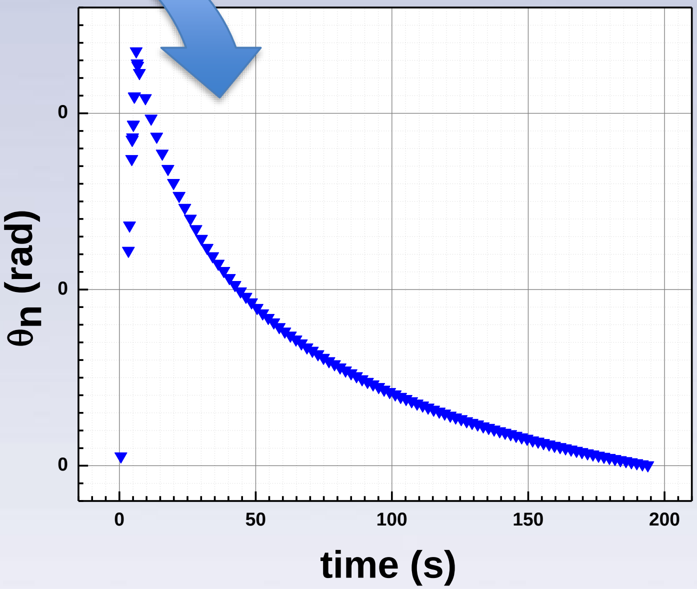
"Negative" peaks

Data analysis. Finding the peaks.

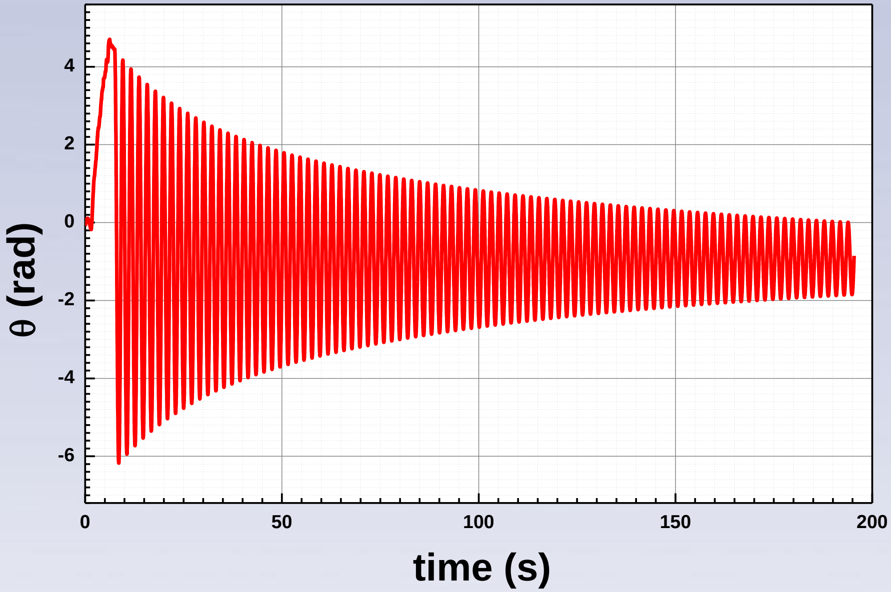
Original Data

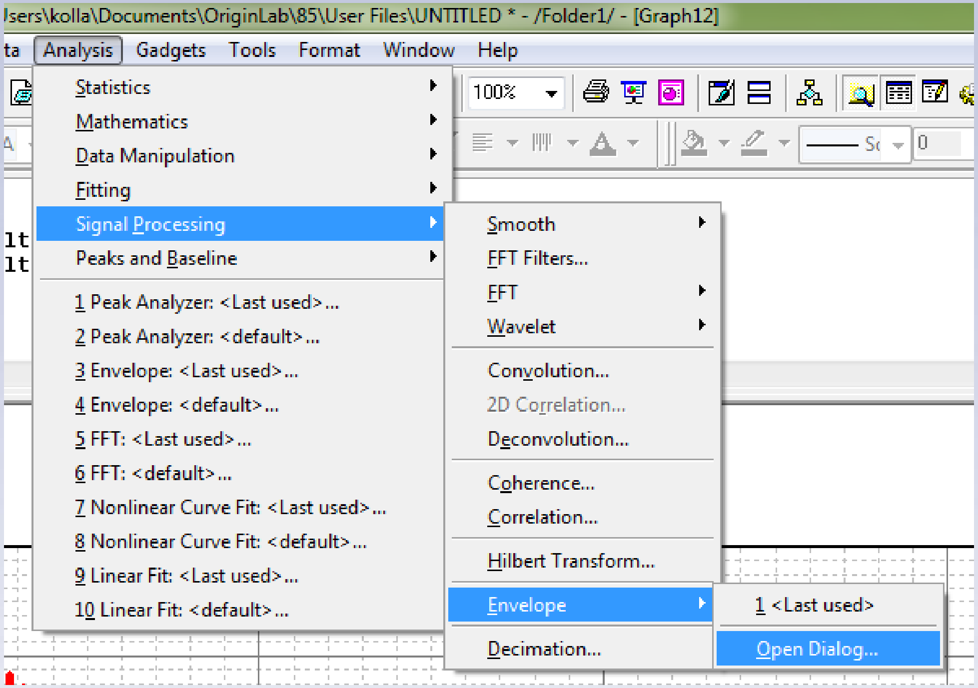


Envelope



Data analysis. Finding the peaks.





2nd Technique: using "*Envelope*" option

Origin will create the worksheet with interpolated (defined for the same x's as the raw data) "envelope" data

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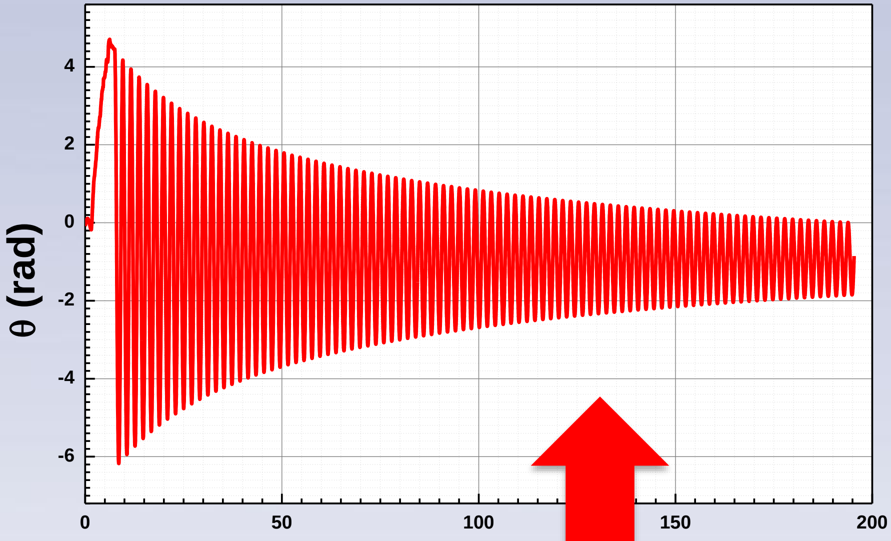
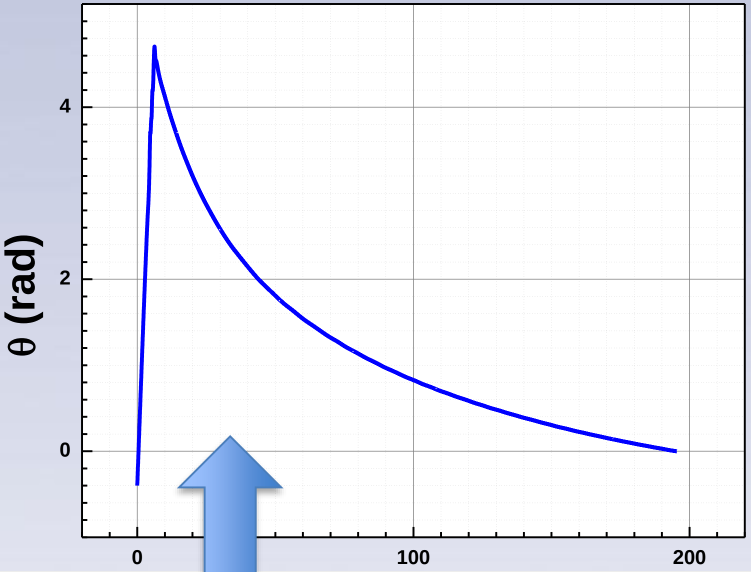
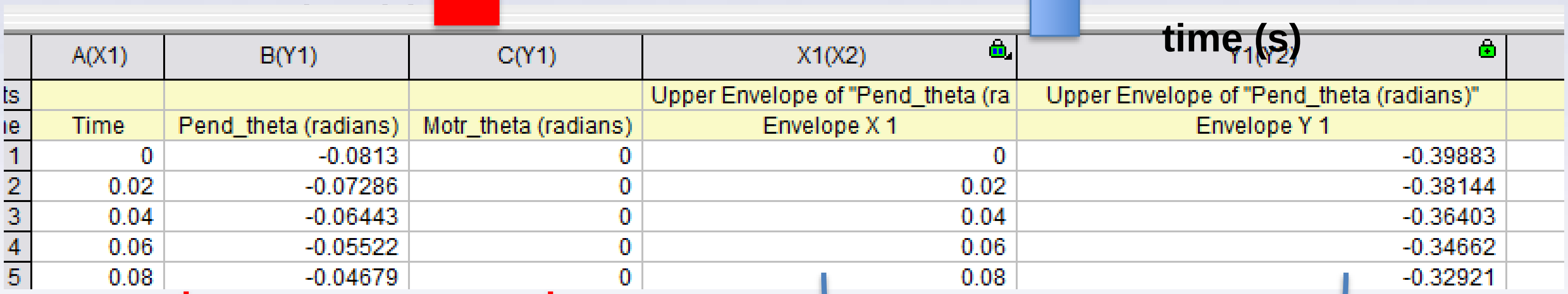
Data analysis. Finding the peaks.







	A(X1)	B(Y1)	C(Y1)	X1(X2)	Y1(Y2)
ts				Upper Envelope of "Pend_theta (ra	Upper Envelope of "Pend_theta (radians)"
ie	Time	Pend_theta (radians)	Motr_theta (radians)	Envelope X 1	Envelope Y 1
1	0	-0.0813	0	0	-0.39883
2	0.02	-0.07286	0	0.02	-0.38144
3	0.04	-0.06443	0	0.04	-0.36403
4	0.06	-0.05522	0	0.06	-0.34662
5	0.08	-0.04679	0	0.08	-0.32921

Original data

Envelope data

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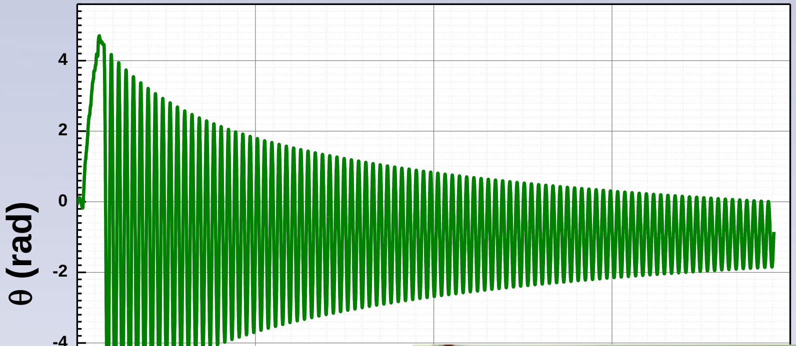
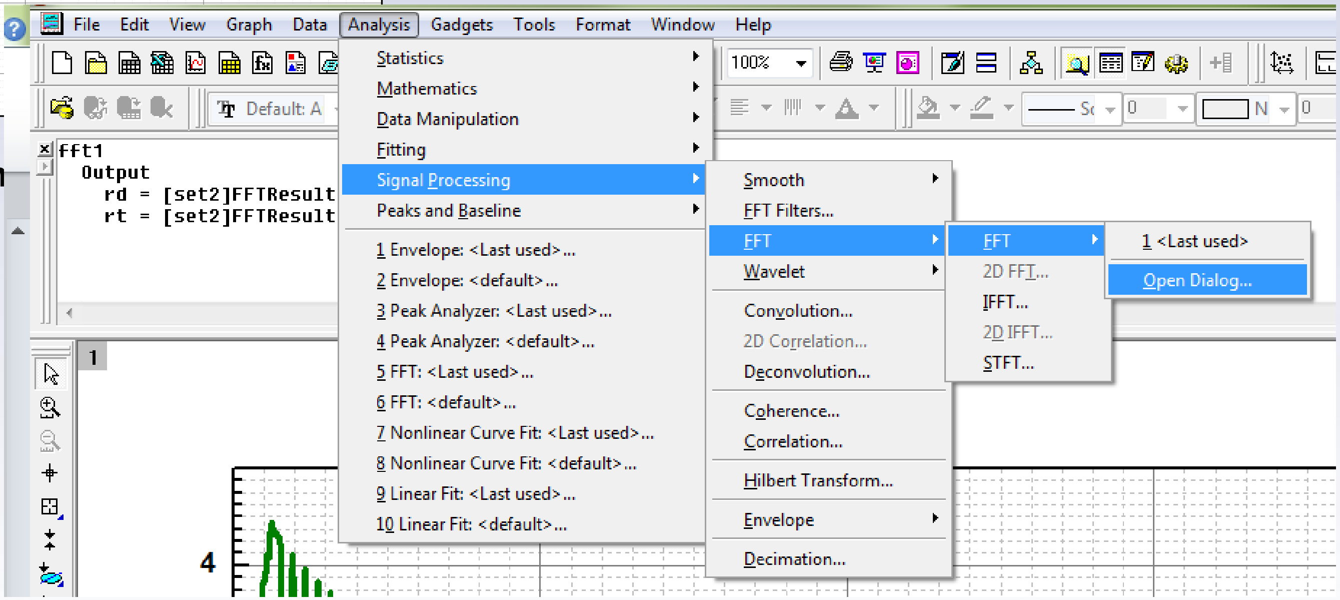
Data analysis. FFT.

All these quasi periodic data can be analyzed using Fast Fourier Transform

Our goal: **find the resonance frequency of the pendulum**



***Origin* window**



```
fft1
  Output
    rd = [set2]FFTResult
    rt = [set2]FFTResult
```

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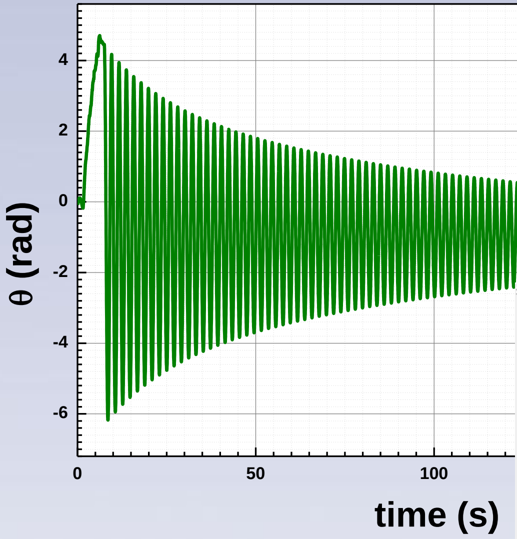
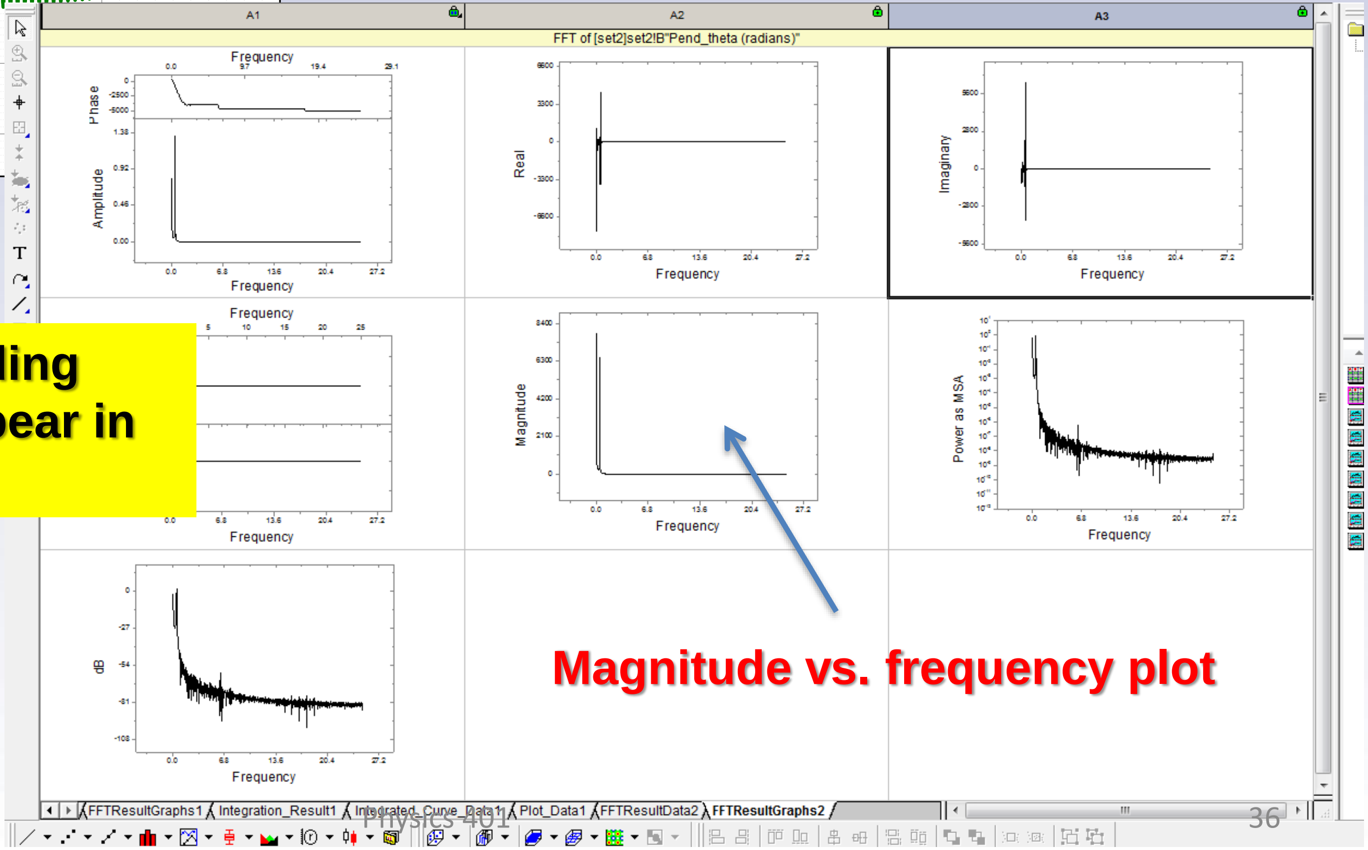
Data analysis. FFT.



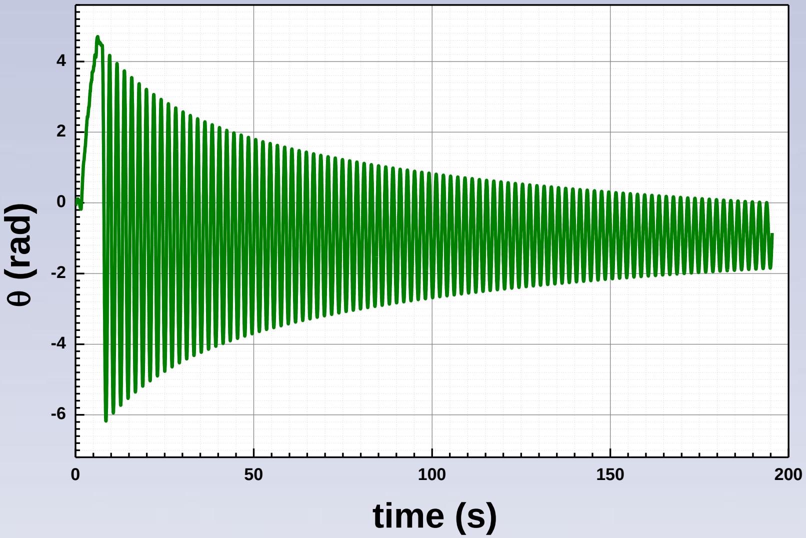
The results of FFT you can find in the same Workbook which contains the raw data

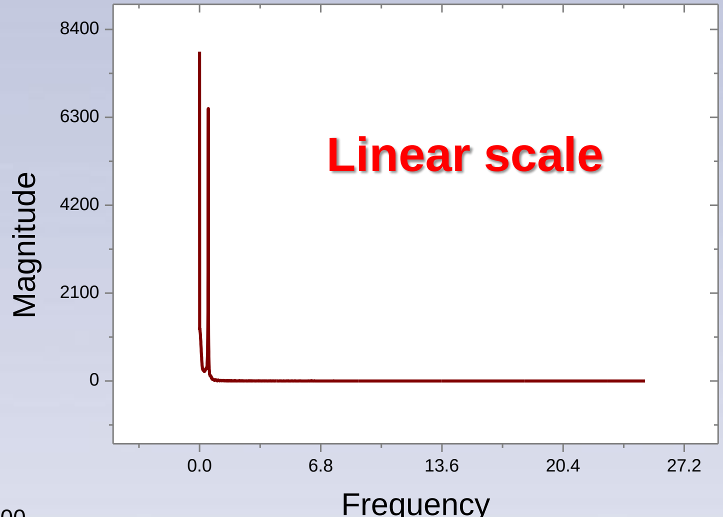


Click on corresponding graph and it will appear in separate window

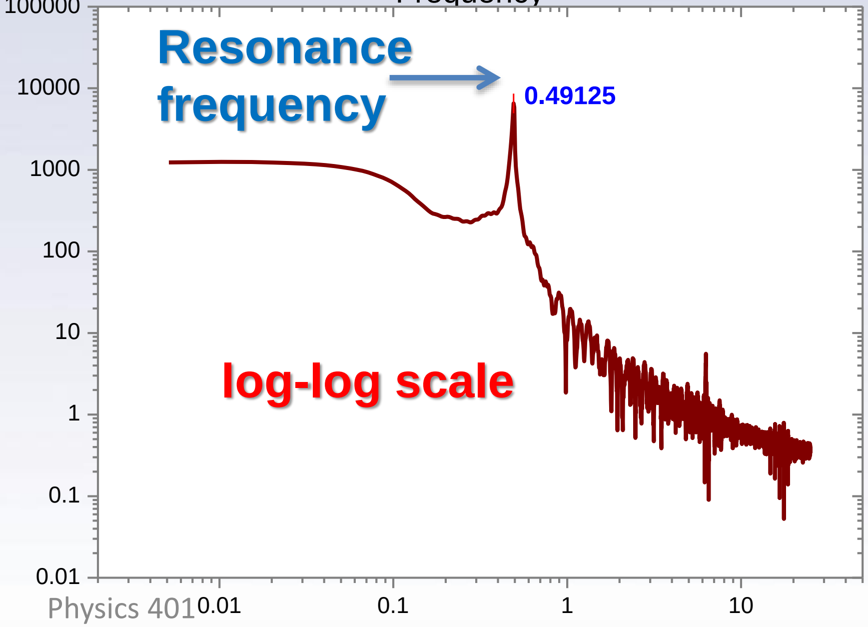
Magnitude vs. frequency plot

Data analysis. FFT.





Linear scale



Resonance frequency

log-log scale

Spectrum better to present in log-log scale

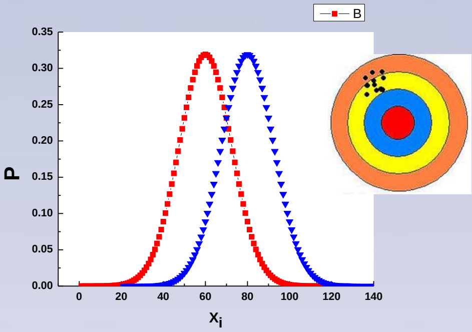
Appendix. Some comments on oil drop experiment error analysis.

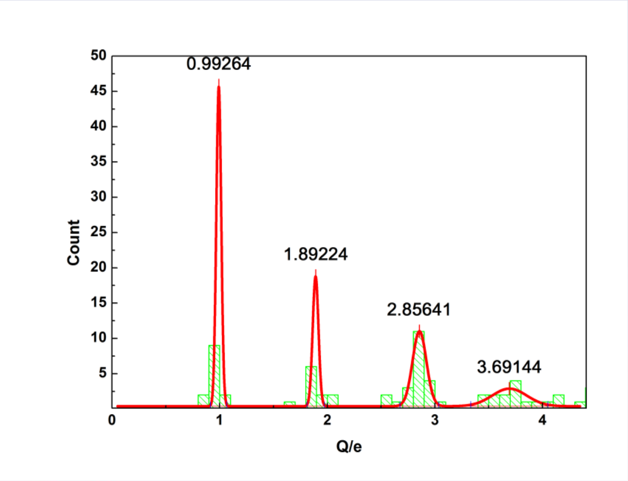
Result of measurement **Systematic error**

$$X_{meas} = X_{true} + e_s + e_r$$

Correct value

Random error





Appendix. Some comments on oil drop experiment error analysis.

Systematic error

$$X_{meas} = X_{true} + e_s + e_r$$

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

$$\boxed{F = \frac{1}{f_c^{3/2}} \approx 1-\left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}}$$

$$\boxed{S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}}$$

$$\boxed{T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)}$$

$$\Delta Q = \sqrt{\left(\frac{dQ}{dF}\right)^2(\Delta F)^2+\left(\frac{dQ}{dS}\right)^2(\Delta S)^2+\left(\frac{dQ}{dT}\right)^2(\Delta T)^2} \approx \sqrt{\left(\frac{dQ}{dS}\right)^2(\Delta S)^2+\left(\frac{dQ}{dT}\right)^2(\Delta T)^2}$$

$$= \sqrt{(FT)^2(\Delta S)^2+(FS)^2(\Delta T)^2} = Q\sqrt{\left(\frac{\Delta S}{S}\right)^2+\left(\frac{\Delta T}{T}\right)^2}$$

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Appendix. Some comments on oil drop experiment error analysis.

Systematic error

$$X_{meas} = X_{true} + e_s + e_r$$

$$\Delta Q \approx Q\sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(\frac{\Delta T}{T}\right)^2}$$

$$\boxed{\frac{\Delta S}{S} = \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta V}{V}\right)^2 + \left(\frac{3}{2}\frac{\Delta x}{x}\right)^2 + \left(\frac{3}{2}\frac{\Delta \eta}{\eta}\right)^2 + \left(\frac{1}{2}\frac{\Delta \rho}{\rho}\right)^2 + \left(\frac{1}{2}\frac{\Delta g}{g}\right)^2} \approx \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{3}{2}\frac{\Delta x}{x}\right)^2}}$$

$$\boxed{\Delta T = \sqrt{\left(\frac{3/2}{t_g^{5/2}} + \frac{1/2}{t_g^{3/2}}\frac{1}{t_{rise}}\right)^2 \Delta t_g^{\,2} + \left(\frac{1}{t_g^{1/2}}\frac{1}{t_{rise}^{\,2}}\right)^2 \Delta t_{rise}^{\,2}}}$$

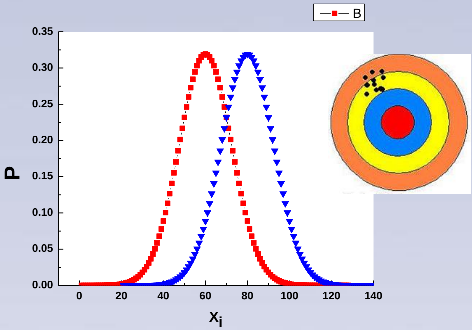
Appendix. Some comments on oil drop experiment error analysis.

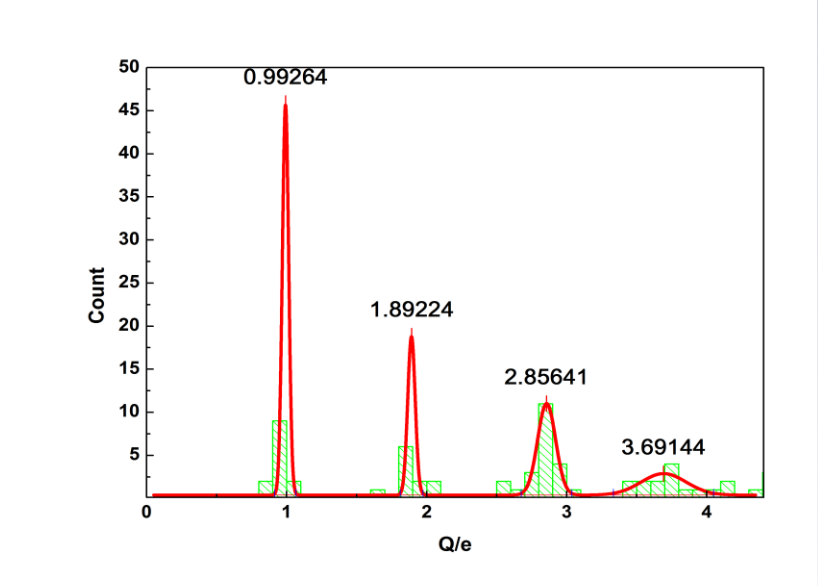
Result of measurement **Systematic error**

$$X_{meas} = X_{true} + e_s + e_r$$

Correct value

Random error





Mean of {x_i}

$$\mu = \frac{1}{N}\sum_{i=0}^{N-1} x_i$$

Standard deviation of of {x_i}

$$\sigma^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}\left(x_i - \mu\right)^2$$

Standard deviation of mean

$$\sigma_X = \frac{\sigma}{N^{1/2}}$$